Topology & Confinement - 3

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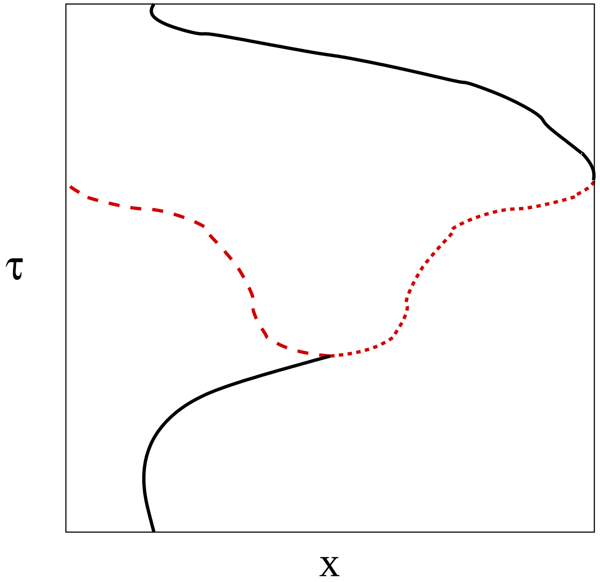
EuroPLEx Online School 2020 - **18 November 2020**

OUTLINE

- Fighting Topological Freezing
- Fighting Rare Topological Fluctuations
- Fighting Discretization Problems with Dynamical Fermions
- Confinement, Deconfinement and Topology

Fighting topological freezing: a few ideas

The dream: tunneling from one topological sector to the other in one step



For the particle on a circumference, one can devise a "cut and paste" move making such a tunneling:

tailor method

C. Bonati, M.D. PRE 98, 013308 (2018), arXiv:1709.10034

General idea

- Find a region of space-time where $Q = \pm 1/2$
- Make a transformation on that region (e.g., charge conjugation) sending $Q \to -Q$
- Glue back the region to the rest of the world $\implies \Delta Q = \pm 1$
- In general, it will lead to large action changes and will be Metropolis-rejected. Works in 1D: $dx/d\tau \to -dx/d\tau$ no idea for higher dimensions

A more flexible method: simulated or parallel tempering

General idea: autocorrelation time of Q changes very rapidly with the lattice spacing. Let the system explore different lattice spacings $a_1, a_2, \ldots a_N$ (different inverse gauge couplings $\beta_1, \beta_2, \ldots \beta_N$) at the same time. Two different implementations:

- **Simulated tempering:** a single system moves dynamically among the different lattice spacings (Metropolis step)
- **Parallel tempering:** N different copies of the same system assigned to N different lattice spacings, then change dynamically the assignement (Metropolis step)

in both cases, decorrelation from large lattice spacings transferred to small lattice spacings. Parallel tempering more time consuming but requires less fine tuning on parameters

A different idea: open boundary conditions

(M. Luscher and S. Schaefer, JHEP 07, 036 (2011), arXiv:1105.4749)

- open boundary conditions $\Longrightarrow$ no homotopy classes

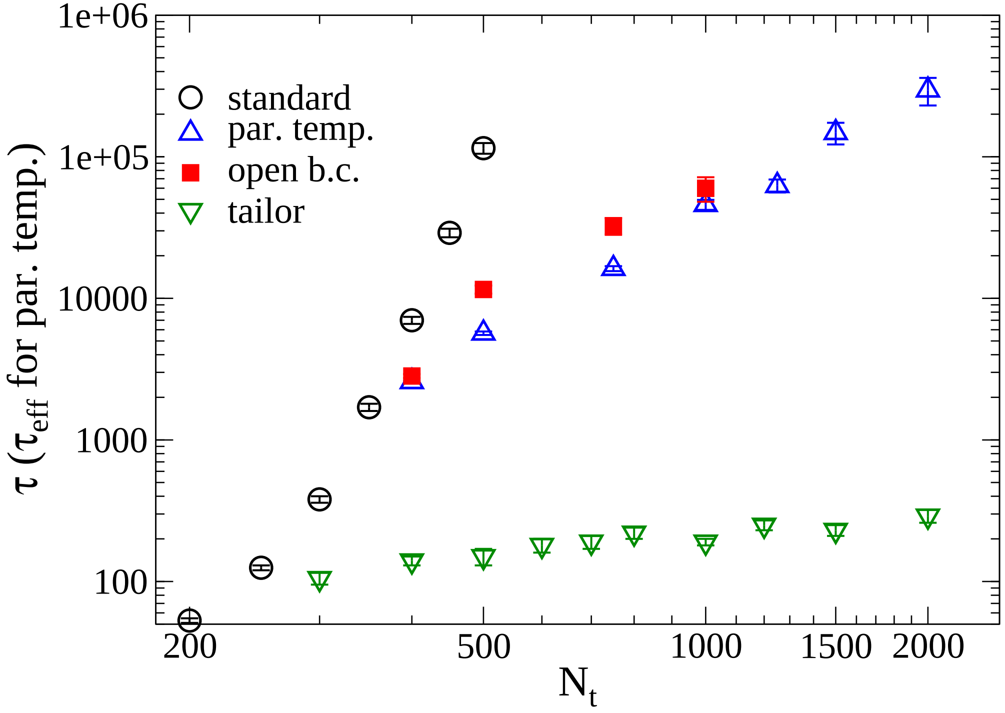
- no barriers, lack of ergodicity disappears

- **Drawback:** larger finite-volume effects, non-integer valued Q, periodicity in θ disappears

- Best way to determine $\langle Q^2 \rangle$ goes through the integral of the two-point function $\langle q(x)q(y) \rangle$. Determination of higher moments would involve n-point correlators, not feasible

How different methods perform for the particle on a circle

C. Bonati, M.D. PRE 98, 013308 (2018), arXiv:1709.10034



- **standard local Metropolis: exponential slowing down** $\tau \sim e^{1/a}$
- **tempering and open b.c.:** $\tau \sim 1/a^2$
- **tailor method:** $\tau \sim$ **flat**

A smart variation: periodic b.c. via open b.c.

(M. Hasenbusch, Phys. Rev. D96, 054504 (2017), arXiv:1706.04443)

- **A parameter fixing the choice b.c. (e.g., the coupling at the boundary) can be made dynamical**

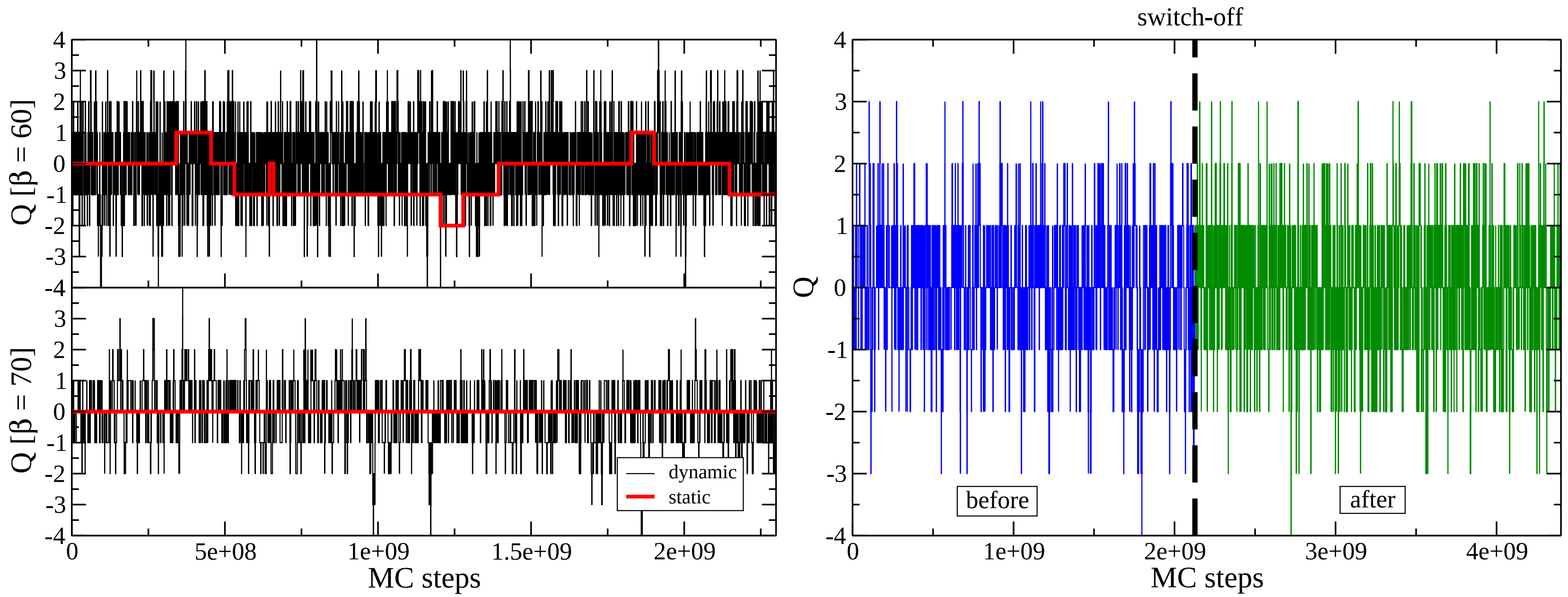
- **In this way, one can simulate periodic and open b.c. at the same time in a parallel tempering setting, which keeps the advantages of both b.c. at the same time**

- **Successfully tested in CP^{N-1} models**

  M. Hasenbusch, Phys. Rev. D96, 054504 (2017), arXiv:1706.04443; M. Berni, C. Bonanno, MD, PRD 100, 114509 (2019), arXiv:1911.03384

A different possibility: local geometry variations

(A. Candido, G. Clemente, M. D'Elia and F. Rottoli, arXiv:2010.15714)



- 2D $U(1)$ gauge theory on a triangulated manifold with locally fluctuating geometry
- Strong improvement induced by non-uniform (rather than varying) geometry
- Heuristic picture: Q fluctuations induced by large holes scattered around the triangulated lattice. Extension to standard $SU(N)$ LGT?

Defeating the rare event problem by a multicanonical approach

C. Bonati, MD, G. Martinelli, F. Negro, F. Sanfilippo and A. Todaro, arXiv:1807.07954

The idea is to modify the probability distribution, by adding a Q dependent potential to the action and then reweight

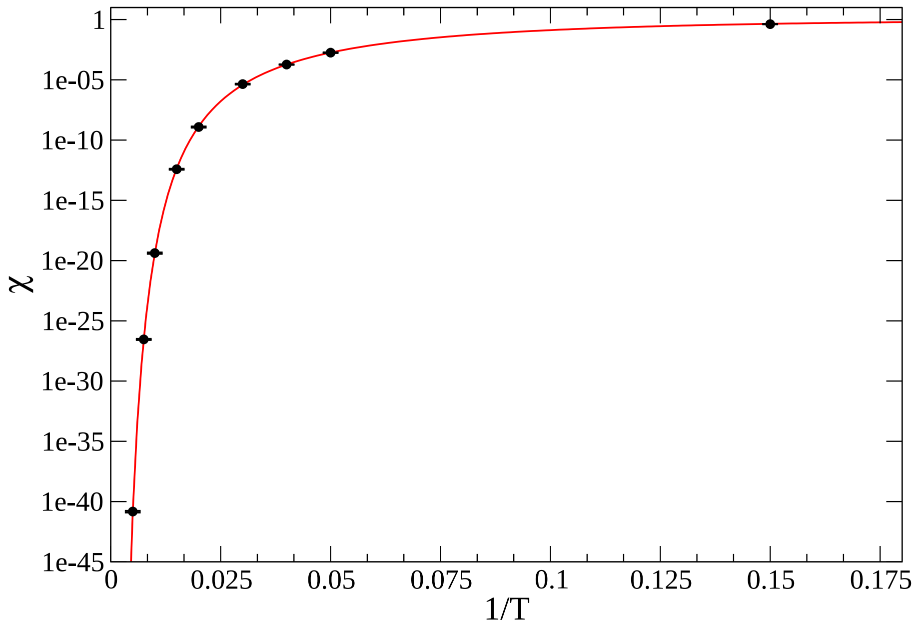
$$\langle Q^2 \rangle = \frac{\int \mathcal{D}U e^{-S_{QCD}}\, Q^2}{\int \mathcal{D}U e^{-S_{QCD}}} = \frac{\int \mathcal{D}U e^{-S_{QCD}-V(Q)}\, Q^2 e^{V(Q)}}{\int \mathcal{D}U e^{-S_{QCD}}} = \frac{\langle Q^2 e^{V(Q)} \rangle_V}{\langle e^{V(Q)} \rangle_V}$$

If $V(Q)$ is chosen so as to enhance high Q configurations, the rare events will be sampled more frequently and then correctly reweighted. The improvement in the statistical error can be impressive.

A similar strategy is adopted in metadynamics, where $V(Q)$ is made dynamical

(Laio, Martinelli, Sanfilippo, arXiv:1508.07270)

This strategy, applied to the case of the path integral on a circle, works impressively: we have been able to determine $\langle Q^2 \rangle$ over 40 order of magnitudes



From C. Bonati, MD, "Topological critical slowing down: seven variations on a toy model", arXiv:1709.10034

Similar strategies have been proposed in the pure gauge case

(P. T. Jahn, G. D. Moore and D. Robaina, arXiv:1806.01162)

We have applied this strategy with the same discretization as before

$N_f = 2 + 1$ QCD, stout improved staggered fermions, a tree-level Symanzik gauge action, physical quark masses

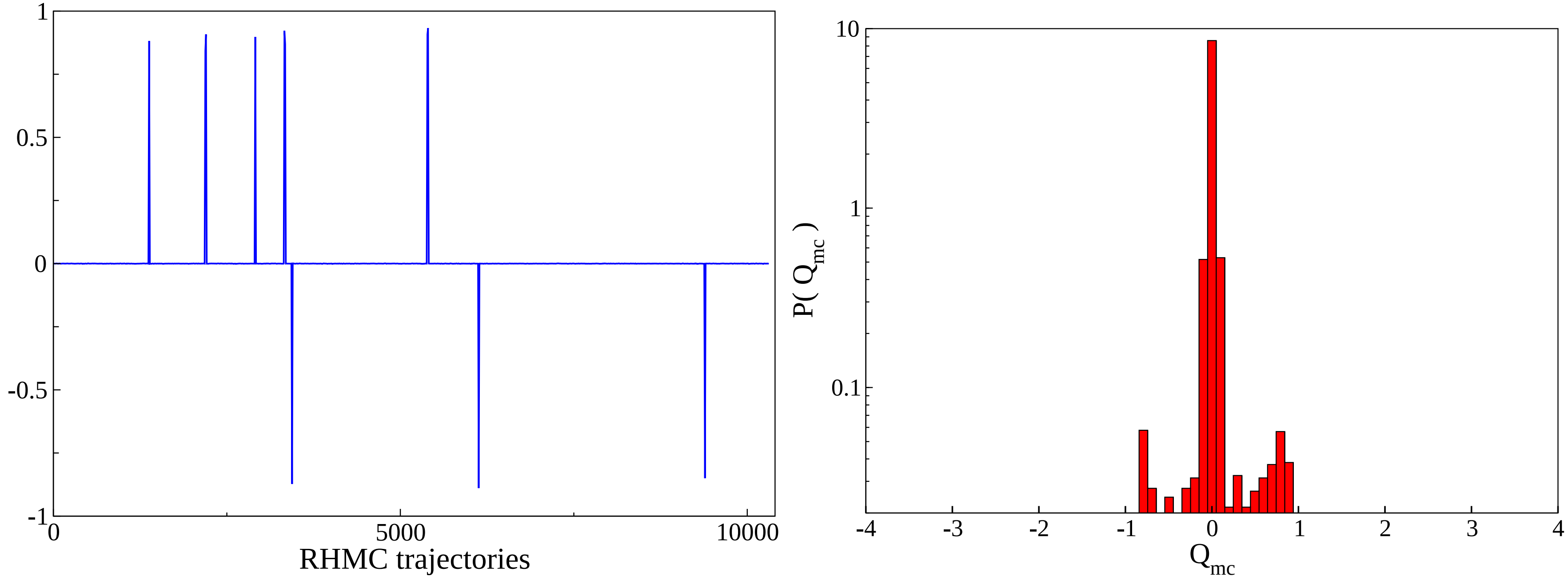
Main technical issues of the numerical implementation:

- the charge Q_{mc} entering the bias potential must be simple enough to permit integration of MD equations with reasonable overhead, have a good overlap with the true topological background Q

  best choice: field theoretic definition after 10-20 steps of stout smearing (30-60% overhead)

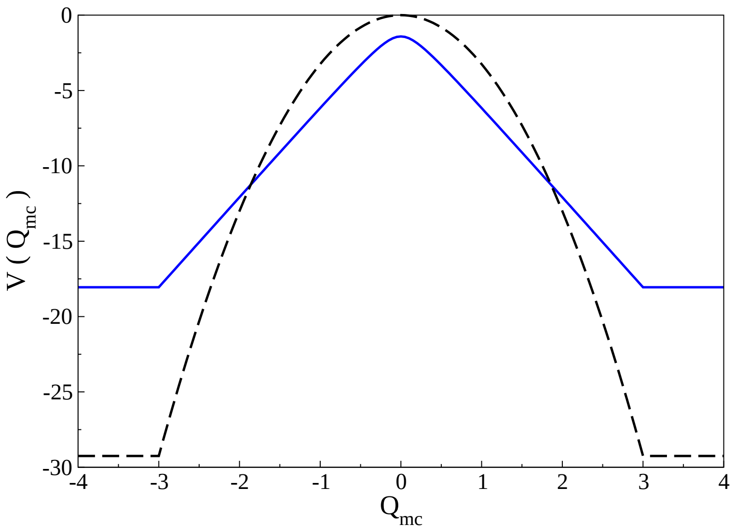
- Reweighting is usually plagued by bad overlap between target and the actual distribution.

  This can happen (and can be avoided) also in our case



MC history of Q and probability distribution of Q_{mc} on a sample run

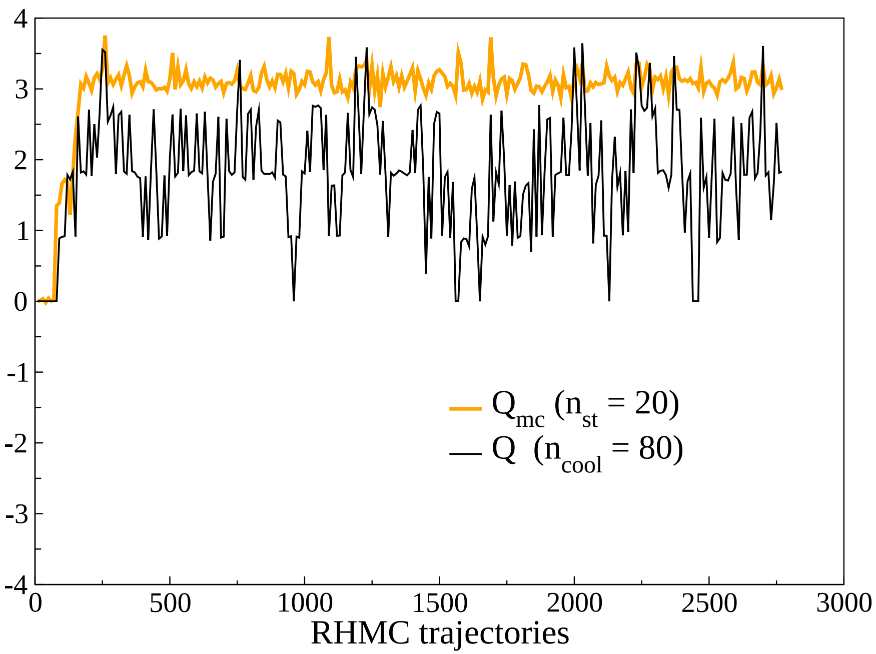
$32^3 \times 8$ **lattice,** $a = 0.0572$ **fm,** $T \simeq 430$ **MeV**

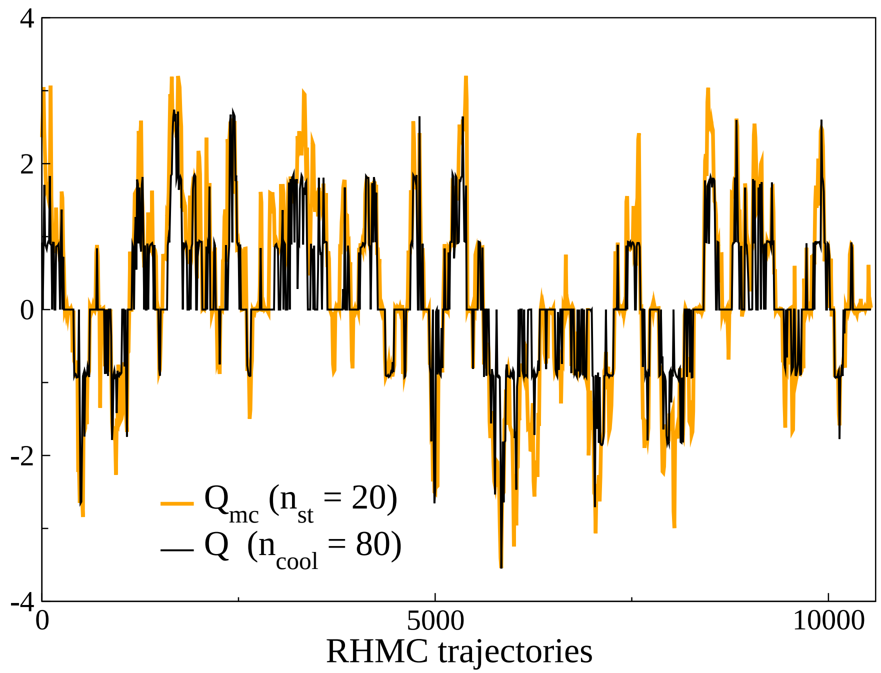


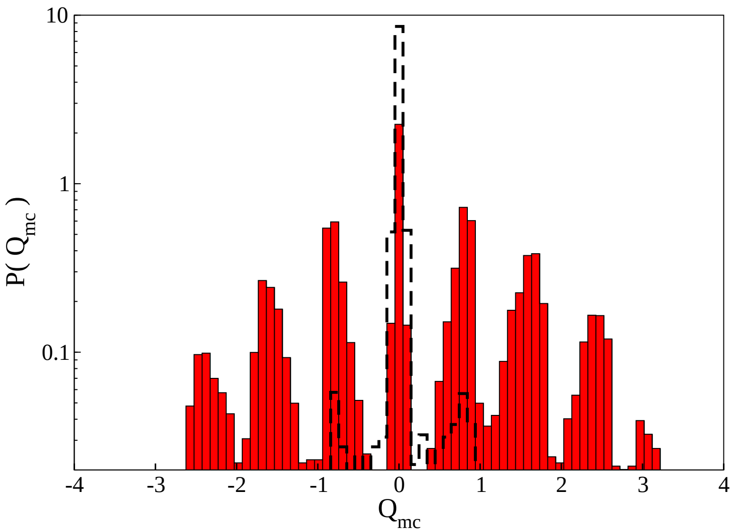
Two possibile choices of the bias potential and the corresponding MC histories:

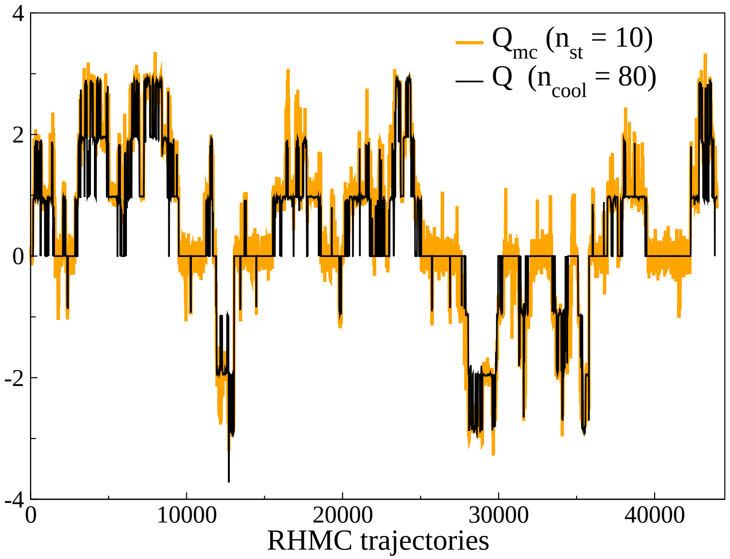
$$V(Q_{mc}) = -a_q\, Q_{mc}^2\,; \quad a_q = 3.25$$

$$V(Q_{mc}) = -\sqrt{(B\, Q_{mc})^2 + C}\,; \quad B = 2,\ C = 6$$









Modified probability distribution of Q_{mc} from the "good" run

reweighted result $a^4\chi = (6.1 \pm 1.1) \times 10^{-8}$

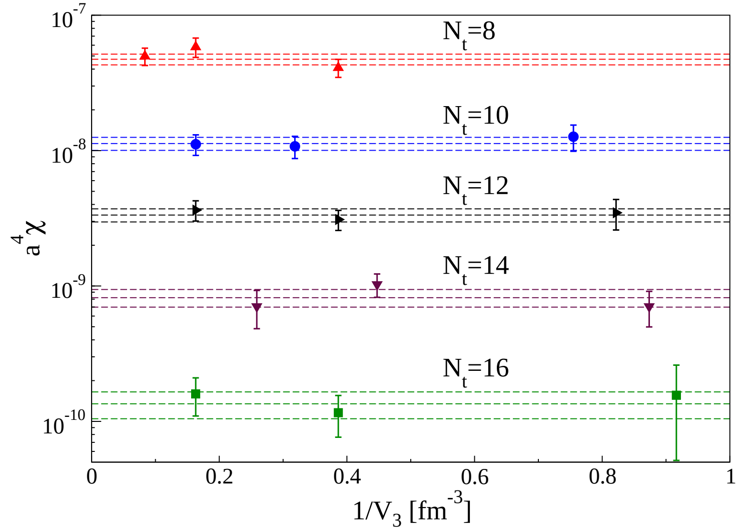
standard method $a^4\chi = (4.1 \pm 1.6) \times 10^{-8}$

taking into account computational overhead (60%), the gain is around 2.5

As the lattice spacing decreases χ drops down and the gain increases

$48^3 \times 16$ **lattice,** $a = 0.0286$, $T = 430$ **MeV**

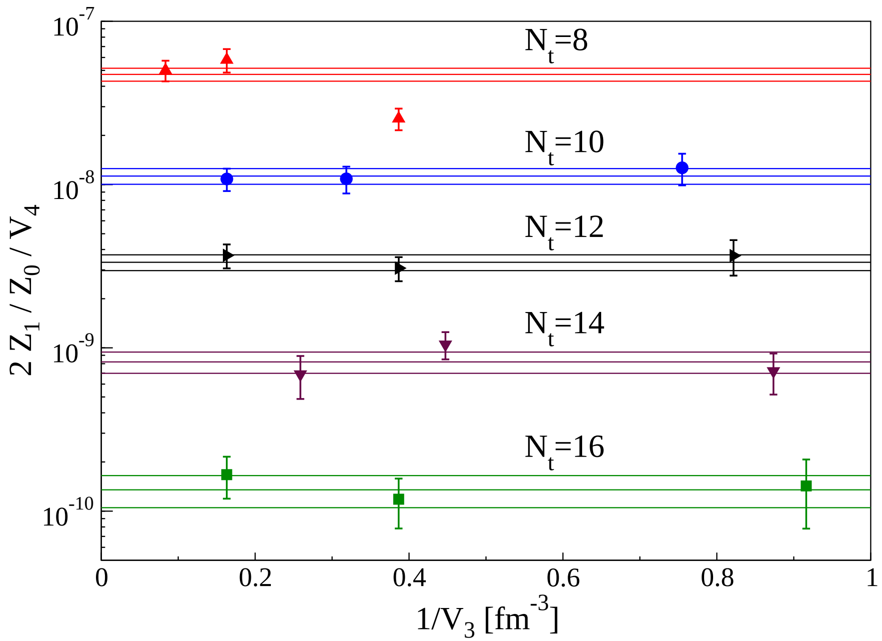
In this case $\langle Q^2 \rangle = 2.1(7) \times 10^{-4}$ **and the estimated gain is** $O(10^3)$**.**

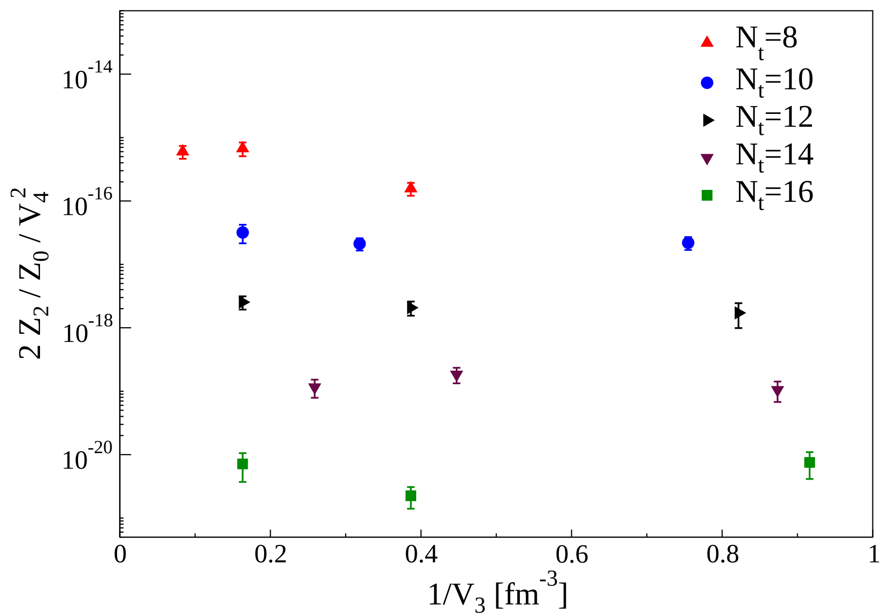


$a^4\chi$ **for various volumes and lattice spacings at** $T = 430$ **MeV**

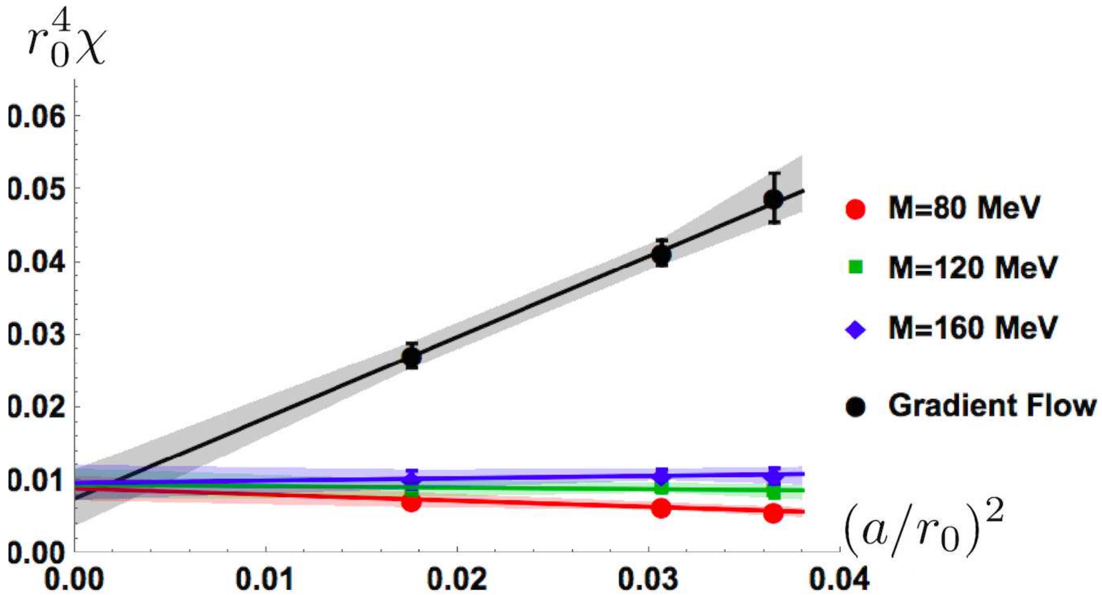
most signal from Z_1/Z_0

however we estimate Z_2/Z_0 **as well: it scales according to DIGA but now the information is significant**





Fighting discretization effects: Spectral Projectors?



- **In numerical simulations with light dynamical quarks, spectral projectors show strongly reduced discretization errors** (C. Alexandrou, A. Athenodorou, K. Cichy, M. Constantinou, D. P. Horkel, K. Jansen, G. Koutsou and C. Larkin, PRD 97, (2018), arXiv:1709.06596)

- **The reason is still unclear, but worth checking with different fermions (in progress)**
  naive personal explanation: same fermion operator for measure as for production effectively cancels, at the measure level, the eccess topological charge?

Confinement, Deconfinement, and θ-dependence

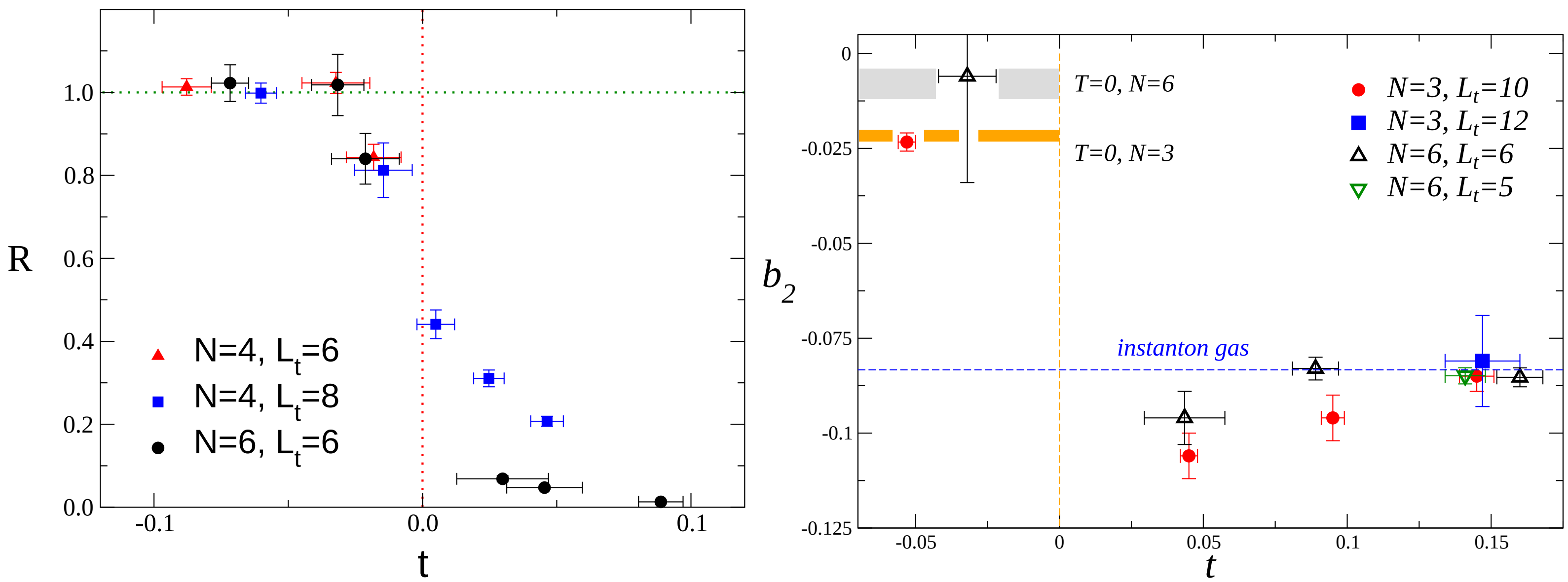
- Confinement emerges as a non-perturbative property of QCD: area law for the Wilson loop (linearly rising potential) and flux tube formation

- Mechanism leading to color confinement still not completely clarified:
  - Dual-superconductor picture (magnetic monopole condensation)
  - Condensation of center vortices
  - ...

- Just for pure gauge theories: a clear association exists between confinement/deconfinement and the exact realization/spontaneous breaking of center symmetry

Emerging picture for $SU(N)$ pure Yang-Mills theories:

- **Confinement (exact realization of center symmetry) is characterized by a θ-dependence compatible with large-N predictions, in particular $F = F(\theta/N)$**
- **Shortly after T_c, topological excitations behave as a dilute non-interacting gas, $F(\theta) \propto (1 - \cos(\theta))$.**



DIGA sets in because of weak coupling or because of a link to center symmetry?

A closer look at the relation between center symmetry and θ-dependence

Is it possible to preserve Z_N center symmetry, even with a small compactification radius (high-T, small coupling), by deforming the pure Yang-Mills action?

M. Unsal and L. Yaffe: PRD 78, (2008) 065035

J.C. Myers and C. Ogilvie: PRD 77, (2008) 125030 (first lattice study)

$$S^{def} = S_{YM} + h \sum_{\vec{n}} |TrP(\vec{n})|^2$$

$SU(3)$: **just one deformation, suppresses large values of $|TrP(\vec{n})|$ locally $\Longrightarrow$ for large enough h, center symmetry is restored even at high-T (small coupling)**

QUESTION: **what happens to θ dependence?**

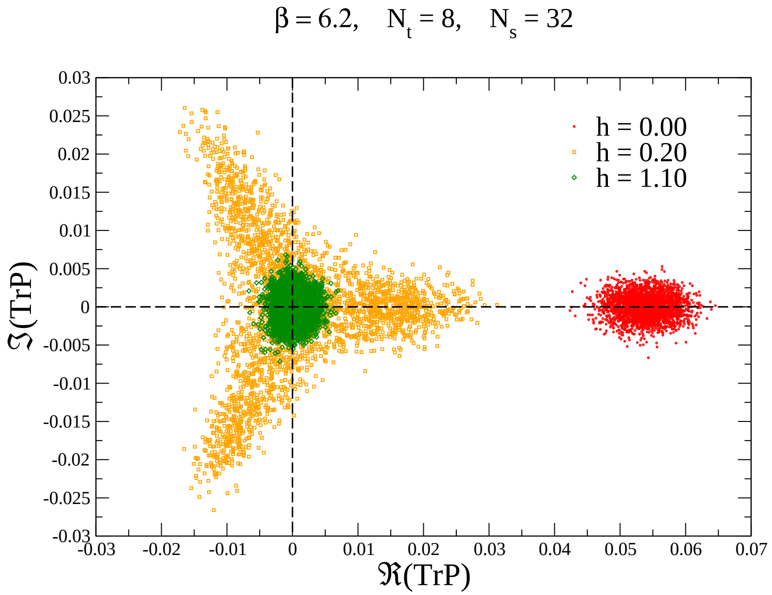
What is DIGA related to? Small coupling or broken center symmetry?

Lattice results:

C. Bonati, M. Cardinali, MD, PRD 98, 054508 (2018), arXiv:1807.06558

C. Bonati, M. Cardinali, MD, F. Mazziotti, PRD 101, 034508 (2020), arXiv:1912.02662

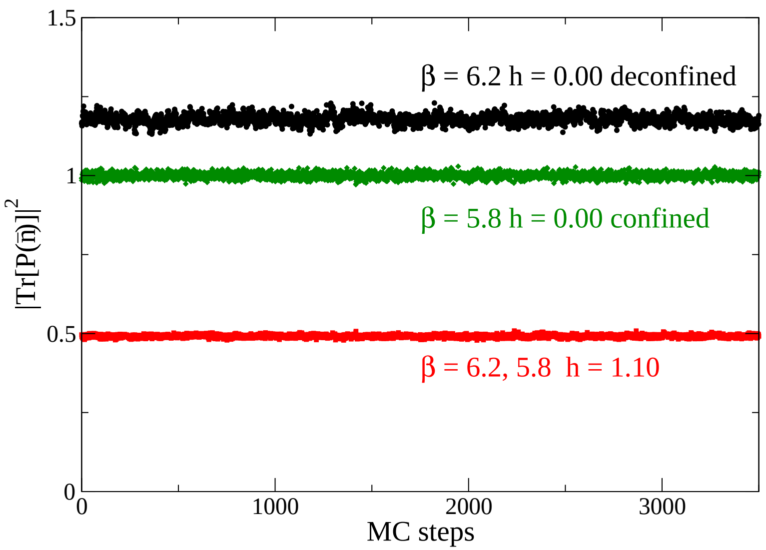
Restoration of Z_3 in the deformed theory



- $T \simeq 1.4\, T_c$, **broken** Z_3 **at** $h = 0$

- **Center symmetry recovered by increasing** h

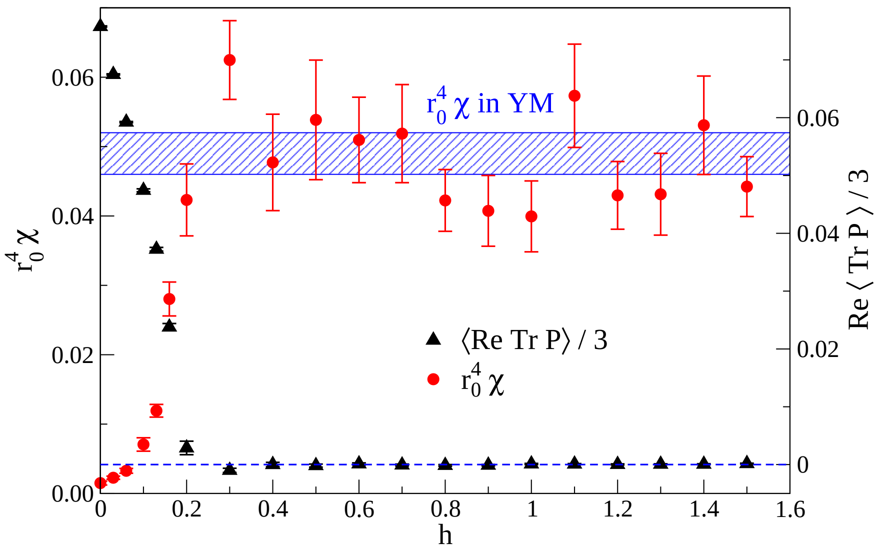
- **Some differences from the standard confined phase emerge looking at the adjoint Polyakov loop**

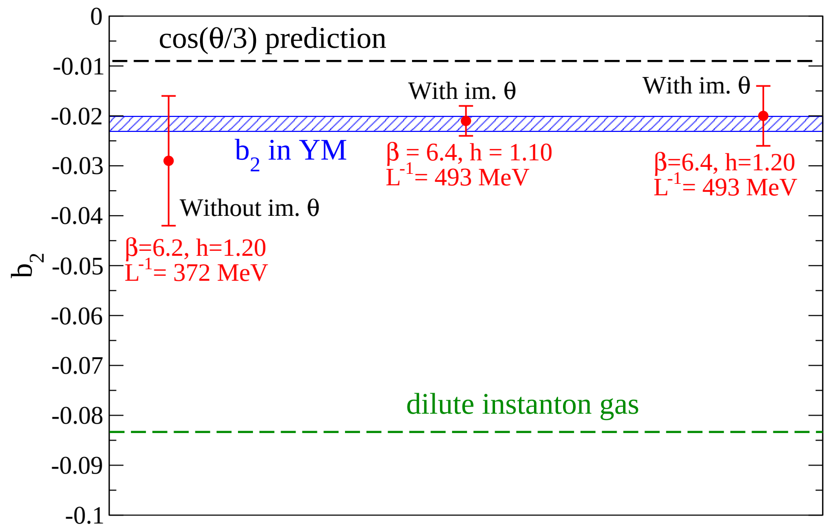


$$P^{adj} = |TrP|^2 - 1$$

a negative value of P^{adj} **means that** $|TrP|$ **tends to vanish locally (point by point).**

For $T < T_c$ **it vanishes by long-range disorder**





θ-dependence seems to be sensible just to the restoration of center symmetry (either locally or by long-range disorder)

- **Left:** the topological susceptibility goes back to its $T = 0$ value
- **Right:** the same happens for b_2.

Notice: semiclassical arguments (Unsal, Yaffe, 2008) predict $b_2 = -1/(12N_c^2)$ (Fractional Instanton Gas Approximation) This is still not observed at the explored L^{-1}

Better insight by going to $N > 3$

C. Bonati, M. Cardinali, MD, F. Mazziotti, in progress

$SU(4)$: **center symmetry has two possible breaking patterns**

$$Z_4 \to \mathrm{Id} \ ; \quad Z_4 \to Z_2$$

Complete restoration of Z_4 requires the vanishing of two traces: P and P^2

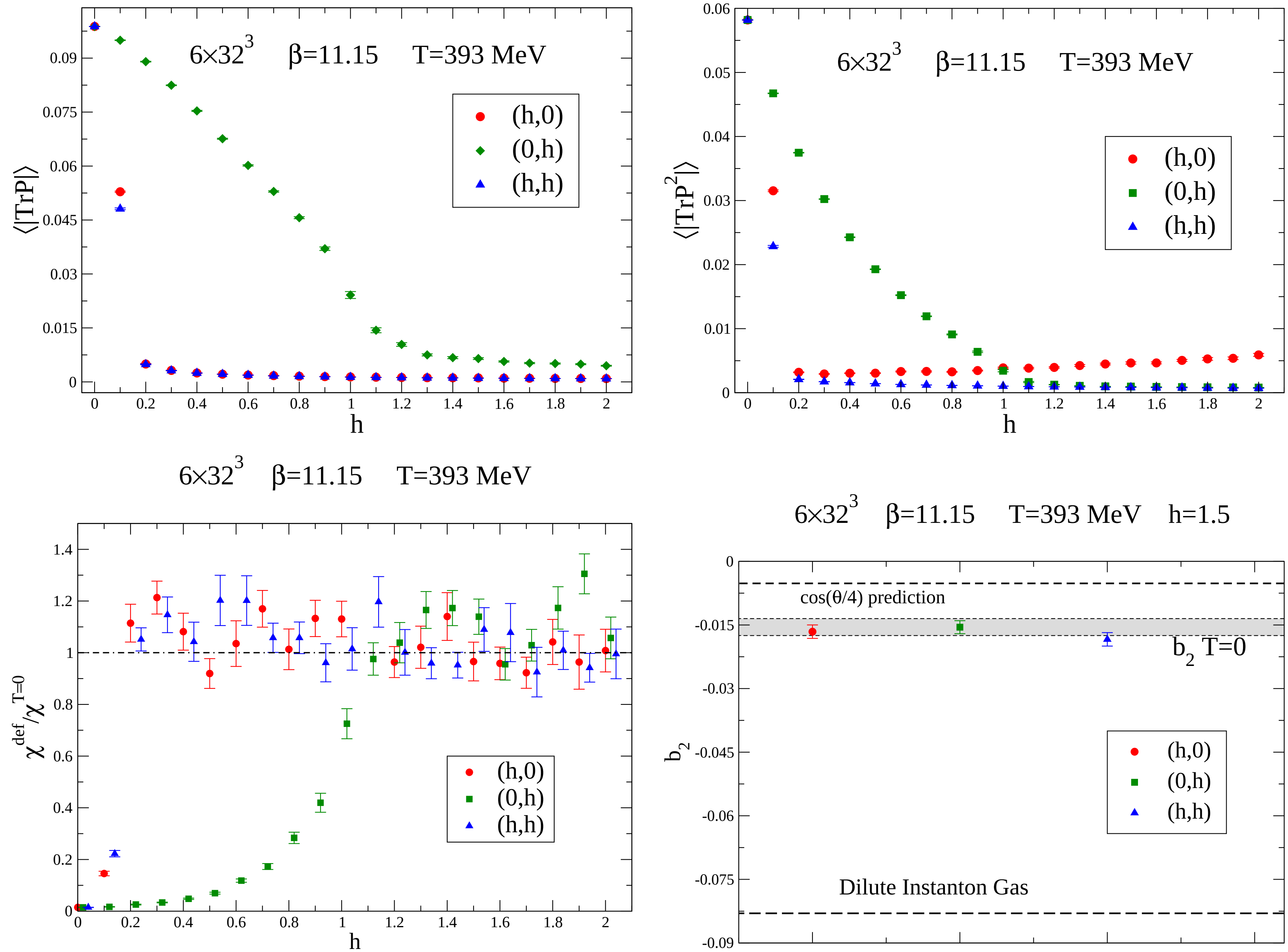
two possible trace deformations to be added to the action

$$S^{def} = S_{YM} + h_1 \sum_{\vec{n}} |TrP(\vec{n})|^2 + h_2 \sum_{\vec{n}} |TrP^2(\vec{n})|^2$$

What about θ-dependence?

Is it sensitive to partial or complete restoration?

ANSWER: θ-dependence back to confined values only for complete restoration



CONCLUSIONS

- The study of θ-dependence and of its relations to confinement and deconfinement is scattered of interesting things yet to explore and fully understand, and of hard algorithmic problems yet to solve

- If you have a chance to work on this topic in the future, you will surely have fun