QCD in finite volume

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EuroPLEx School, fall 2020

Lecture 4

Outline, lecture 4

- **resonances** in one-channel scattering of two spin-less particles

- construction of **interpolators** for scattering of two spin-less particles

- **coupled-channel scattering**

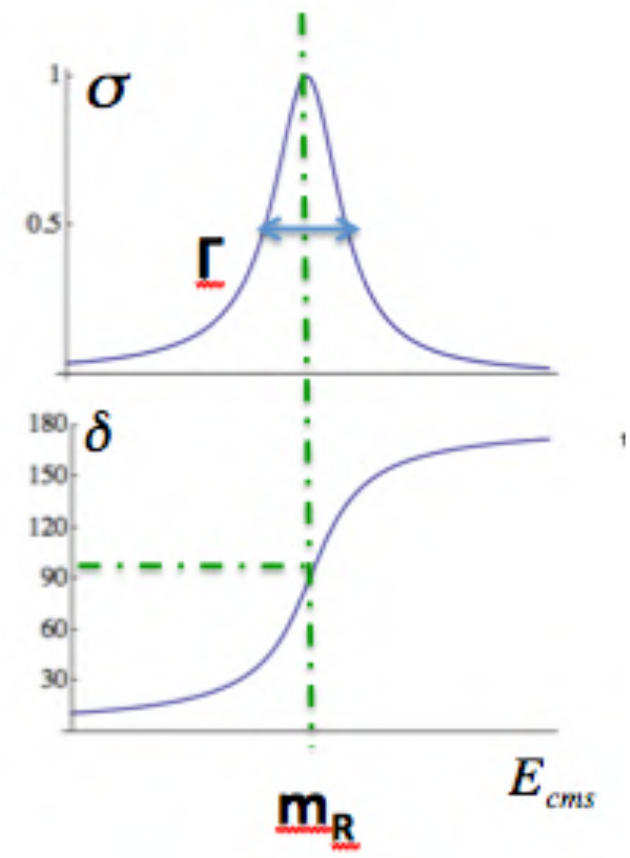
- **scattering of hadrons with spin**

Resonances

that strongly decay only to one channel $H_1 H_2$

where $H_{1,2}$ are spinless hadrons

Reminder on resonances



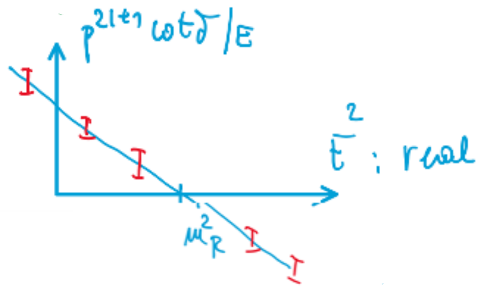
BW

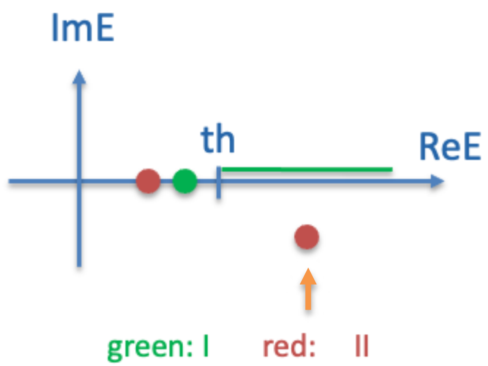
$$T(E_{cm}) = \frac{1}{p}\,\frac{E\;\Gamma(E_{cm})}{m_R^2 - E_{cm}^2 - i\;E_{cm}\;\Gamma(E_{cm})} = \frac{1}{p\cot\delta_l - ip}$$

$$\Gamma(E_{cm}) = g^2\,\frac{p^{2l+1}}{E_{cm}^2}$$

$$\frac{p^{2l+1}\cot\delta}{E_{cm}} = \frac{m_R^2 - E_{cm}^2}{g^2}$$

BW



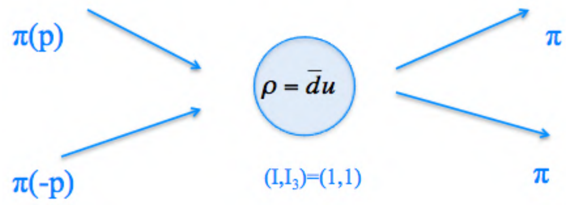


pole of T away from real-axes

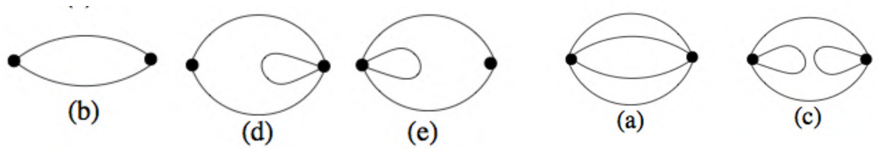
on Riemann sheet II

$\pi\,\pi$ scattering in ρ-channel, $J^{PC}=1^{--}$, I=1

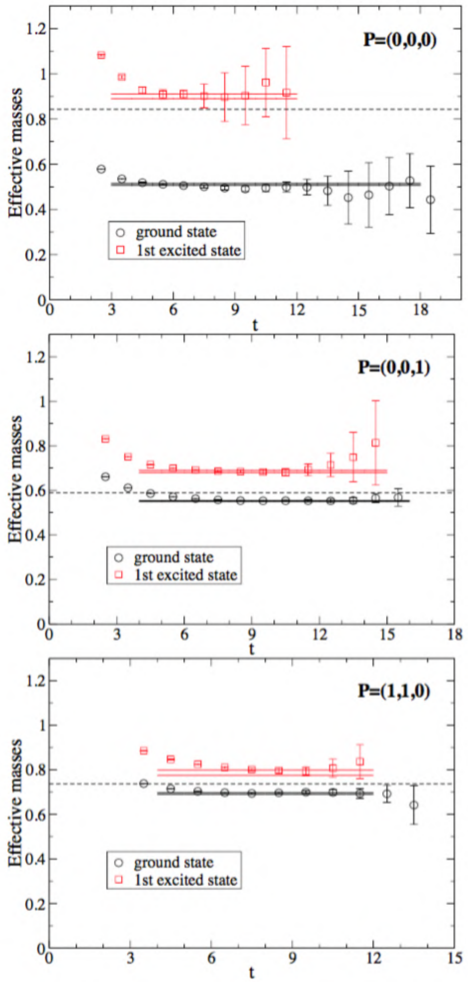
Lang, Mohler, S.P., Vidmar
PRD 2011, m_π=266 MeV, Nf=2
single volume N_L=16



$$C_{jk}(t) = \langle 0 | O_j(t)\, O_k^+(0) | 0 \rangle, \quad O = \bar{q}q,\ (\bar{q}q)(\bar{q}q) = \pi\pi, \quad 1^{--}$$



eigen-energies for NL=16



interpolators

$\pi\pi$

$\bar{q}\ \Gamma\ q\ (000)$ → always several of those

$\pi(-001)\pi(001)$

↓ mom. proj.

$\pi\pi$

$\bar{q}\ \Gamma\ q\ (001)$

$\pi(000)\pi(001)$

$\pi\pi$

↑

dashed lines: E^{ni}

$\bar{q}\ \Gamma\ q\ (110)$

$\pi(000)\pi(110)$

note: En have to be determined so accurately that energy shifts can be extracted reliably

$\pi\ \pi$ scattering in ρ-channel, $J^{PC}=1^{--}$, I=1

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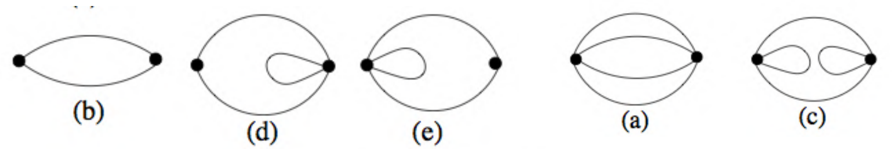
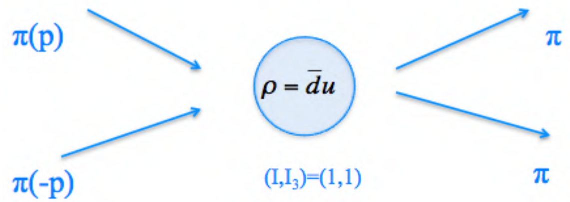
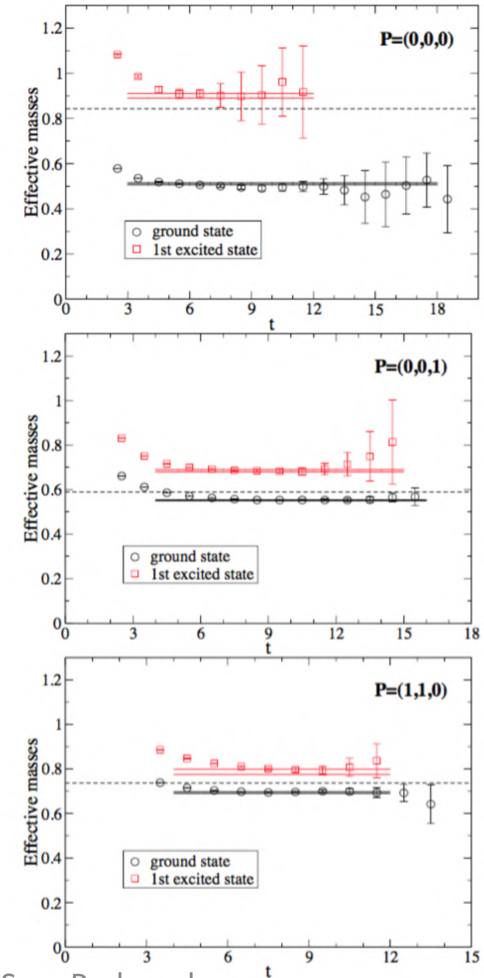




eigen-energies for NL=16

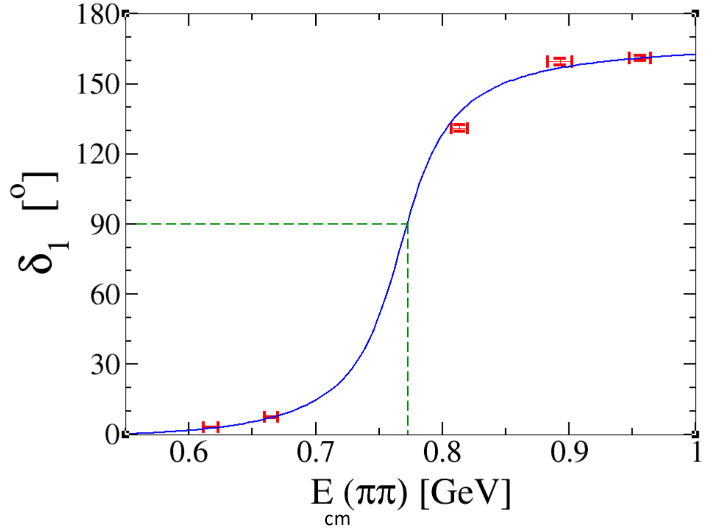


$E_{cm} \rightarrow \delta(E_{cm})$

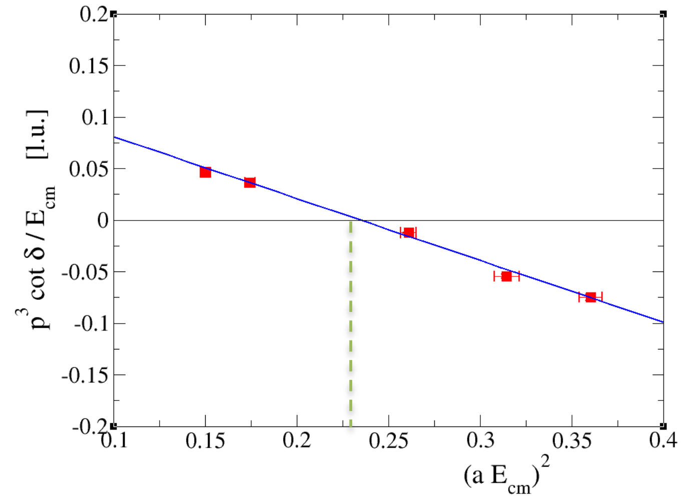
Luscher's relation

for P=0, P ≠ 0

$$\left.\begin{aligned}\Gamma(E_{cm}) &= g^2\,\frac{p^{2l+1}}{E_{cm}^2} \\ \frac{p^{2l+1}\cot\delta}{E_{cm}} &= \frac{m_R^2 - E_{cm}^2}{g^2}\end{aligned}\right\}\ \text{l=1}$$

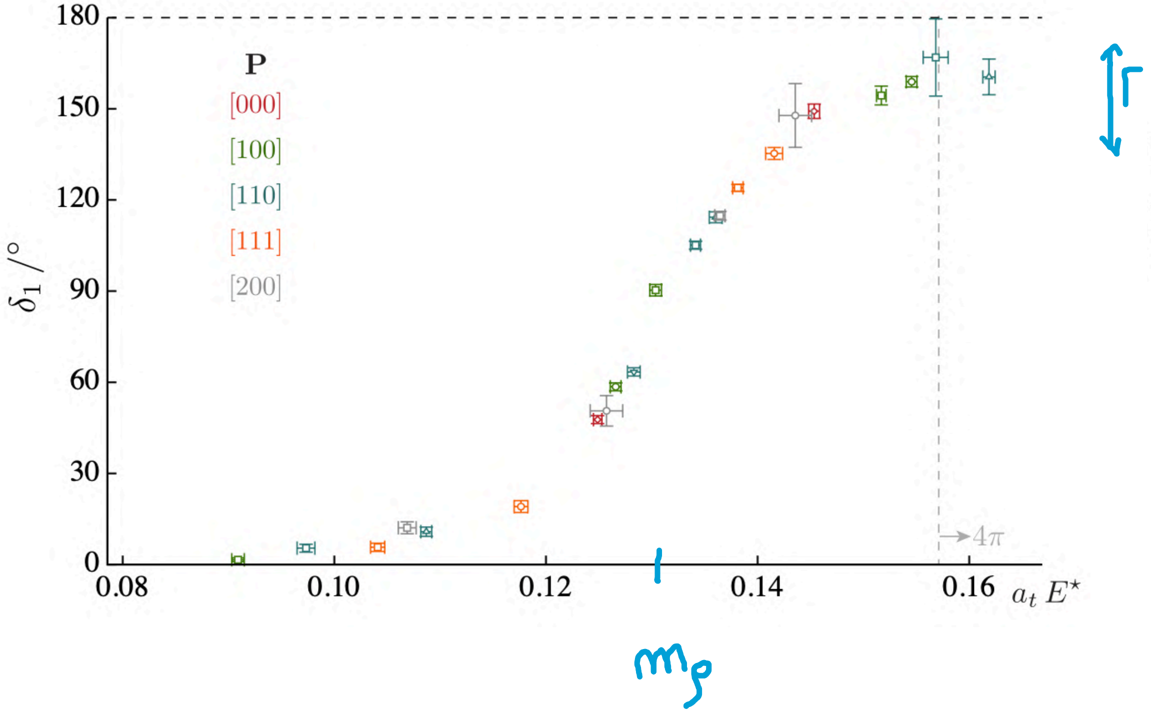


ρ meson	Mass [MeV]	$g_{\rho\pi\pi} \equiv \sqrt{6\pi}\, g$
Lat	772 ± 6 ± 8	5.61 ± 0.12
Exp.	775	5.97

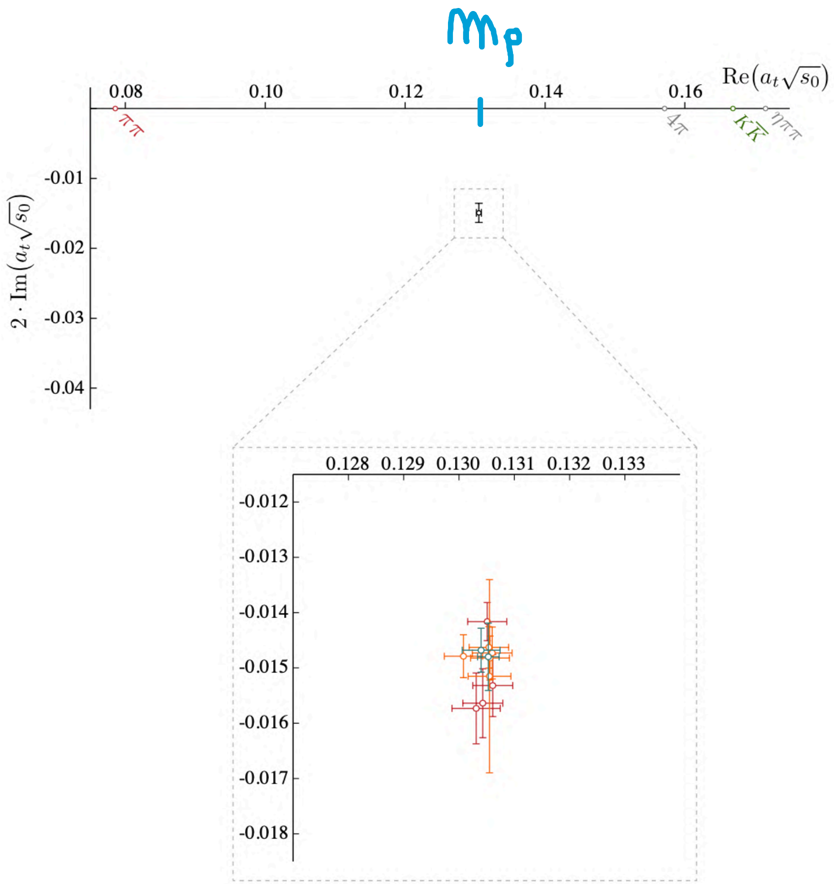


E_{cm}=772 MeV

$\pi\ \pi$ scattering in ρ-channel, $J^{PC}=1^{--}$, I=1



location of pole in complex E plane

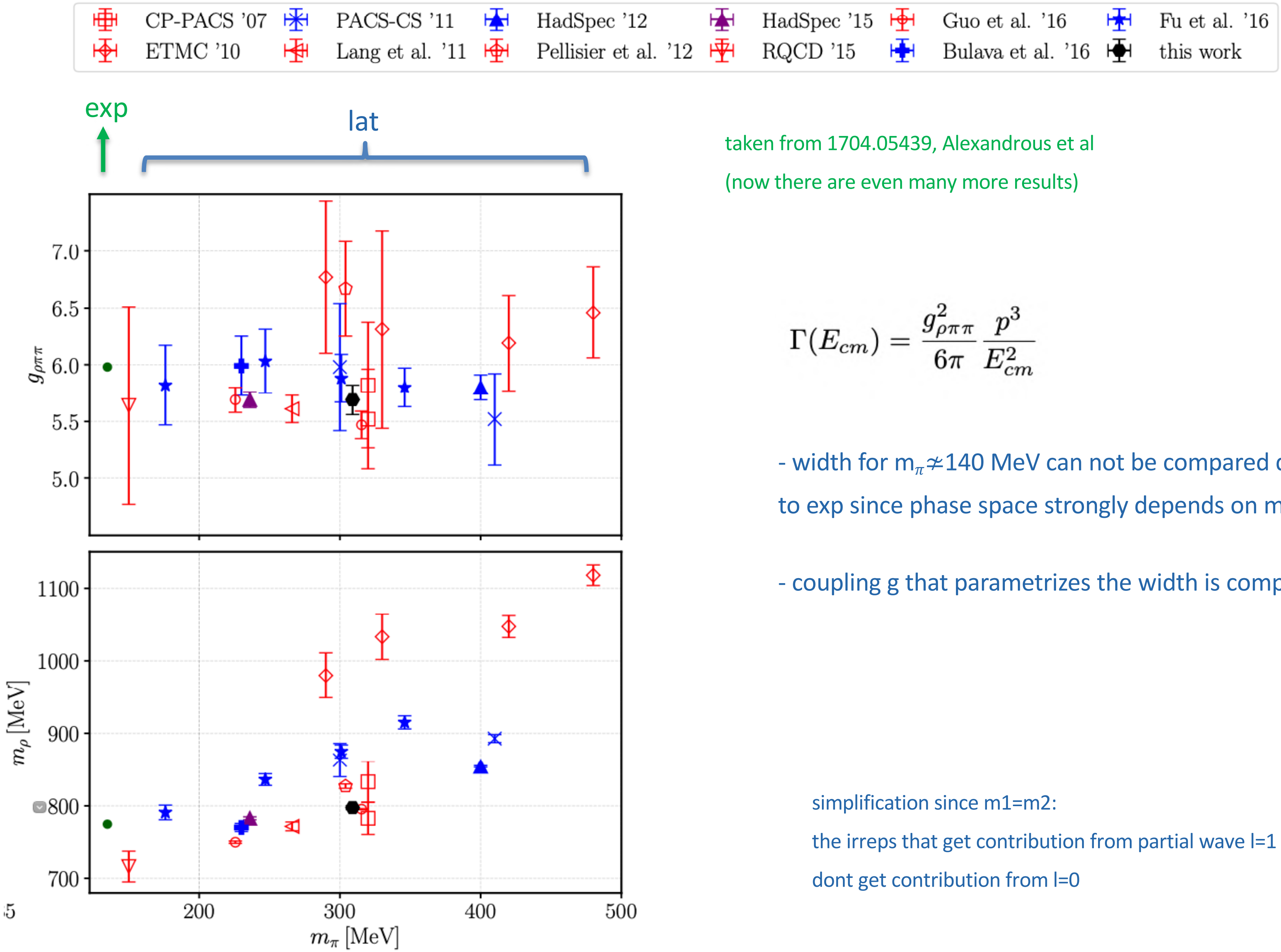


HSC m_π=236 MeV, several NL

plots from 1706.06223

various parametrizations

ρ-resonance is the only resonance that has been extracted by many lattice groups



taken from 1704.05439, Alexandrous et al

(now there are even many more results)

$$\Gamma(E_{cm}) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{p^3}{E^2_{cm}}$$

- width for $m_\pi \neq 140$ MeV can not be compared directly to exp since phase space strongly depends on m_π

- coupling g that parametrizes the width is compared

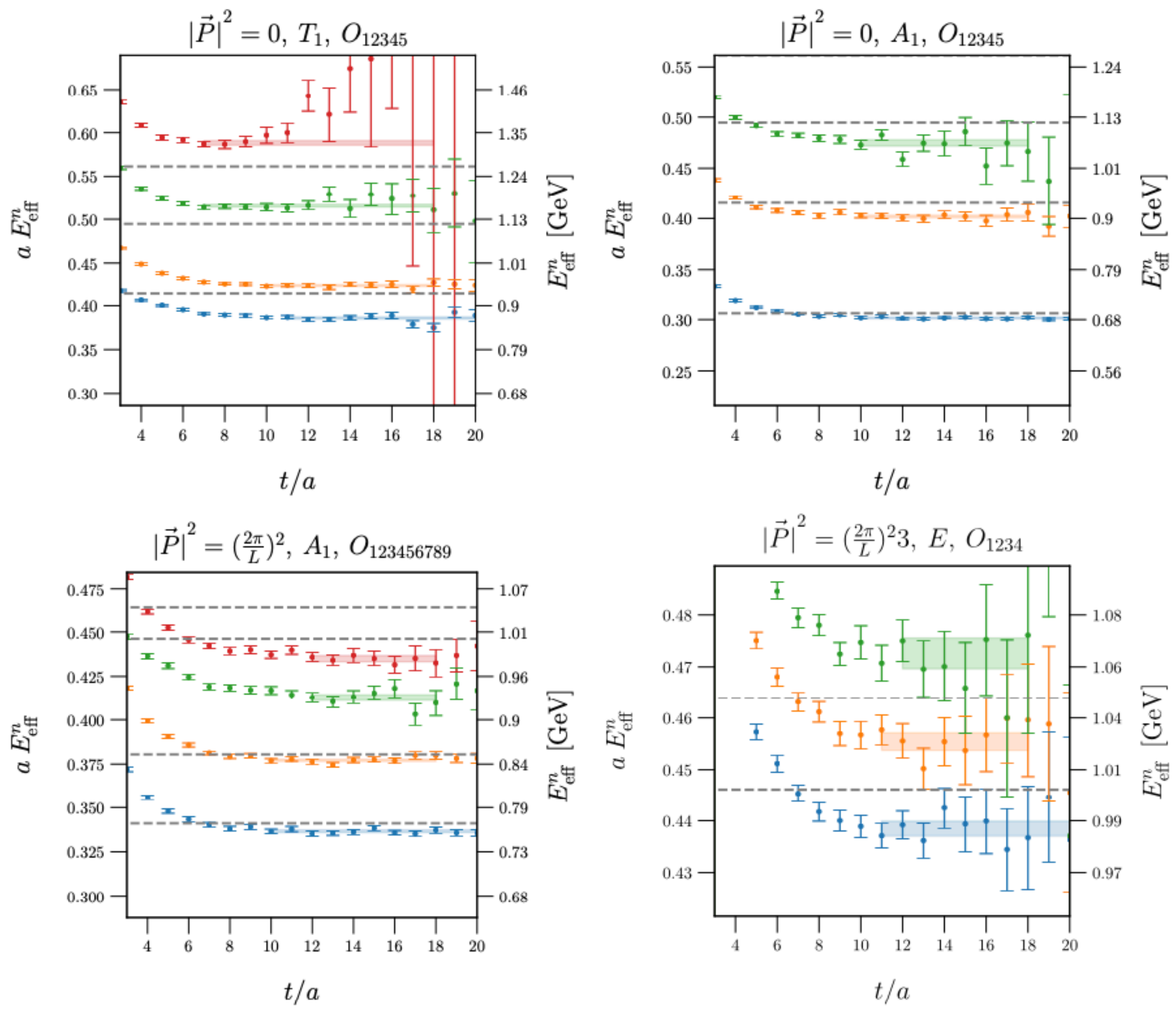
simplification since m1=m2:

the irreps that get contribution from partial wave l=1

dont get contribution from l=0

K π scattering: resonances K* and κ

Rendon et al. 2006.14035



eigen-energies

Reminder: if only partail wave l contributes to a given irrep

$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

quantization condition (Luscher's equation)
for a given irrep

$$1 + i\ \mathcal{M}(E_{cm})_{l,l}\ \mathcal{G}(E_{cm})_{ll} = 0$$

$$\mathcal{M}(E_{cm})_{l,l} = \frac{i}{\mathcal{G}(E_{cm})_{ll}}$$

G is a known function

E_{cm}^{lat} -> $M_{ll}(E_{cm})$

K π scattering: resonances K* and κ

Rendon et al. 2006.14035

challence for P> 0 since mK $\neq$ mπ:

partial waves l = 0 and l = 1 contribute to certain irreps (for example A1)

$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

quantization condition (Luscher's equation)

for a given irrep

M is diagonal for

scat of particles

with S=0

$$M = \begin{vmatrix} M_{\ell=0} & 0 \\ 0 & M_{\ell=1} \end{vmatrix}$$

$$G = \begin{vmatrix} G_{00} & G_{01} \\ G_{10} & G_{11} \end{vmatrix}$$

at given E_{cm} the quantization condition gives only one relation: impossible to extract both

$$f[\mathcal{M}_0(E_{cm}), \mathcal{M}_1(E_{cm})] = 0$$

$$\mathcal{M}_0(E_{cm}), \mathcal{M}_1(E_{cm})$$

rescue: parametrization of M(E)

general idea suggested by

Doring, Meissner, Oset, Rusetsky 1205.4838

$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

quantization condition

M is diagonal for

scat of partticles

with S=0

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{l=0} & 0 \\ 0 & \mathcal{M}_{l=1} \end{pmatrix} \qquad \mathcal{G} = \begin{pmatrix} \mathcal{G}_{00} & \mathcal{G}_{01} \\ \mathcal{G}_{01} & \mathcal{G}_{11} \end{pmatrix}$$

$$\mathcal{M}_l(E_{cm}, C_l^i) = \frac{8\pi E_{cm}}{p \cot\delta_l(p) - ip}$$

1. parametrize M_l as a function of E or p via some parameters (C)

   for example effective range exp.

$$p^{2l+1} \cot\delta_l(p) = C_l^0 + C_l^1\ p^2 + C_l^2\ p^4$$

2. parameters C chosen such that det[]=0 at $E_{cm} = E_{cm,\ n}^{lat}$

$$\det[1 + i\ \mathcal{M}(E_{cm}, C_l^i)\ \mathcal{G}(E_{cm})]_{L,\vec{P},\Lambda} = 0$$

for all L,P, Λ=irrep studied on the lattice

3. previous point is hard to satify exacly, so one searches for C that satisfy it best by minimizing chi2 below

$$\det[1 + i\ \mathcal{M}(E_{cm}, C_l^i)\ \mathcal{G}(E_{cm})]_{L,\vec{P},\Lambda} = 0 \longrightarrow E_{cm,\ n}^{model}\ (C)$$

$$\chi^2(C) = \sum_{a,b}\ [E_{cm,a}^{lat} - E_{cm,a}^{model}(C)]\ \mathrm{cov}_{ab}^{-1}\ [E_{cm,b}^{lat} - E_{cm,b}^{model}(C)]$$

sum a,b over

all discrete energy levels n

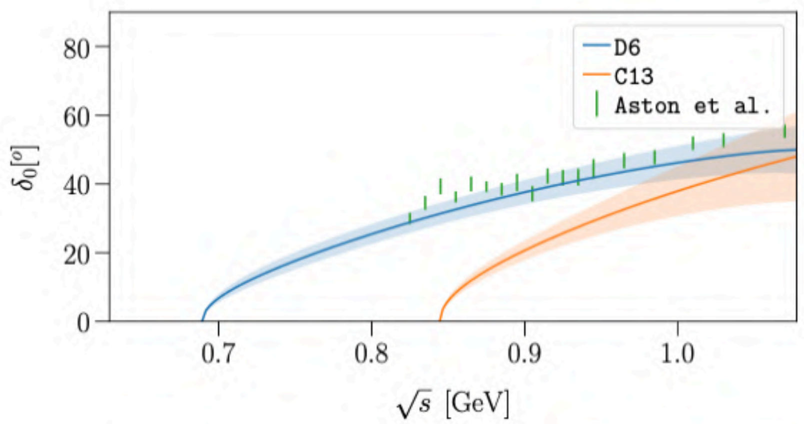
and

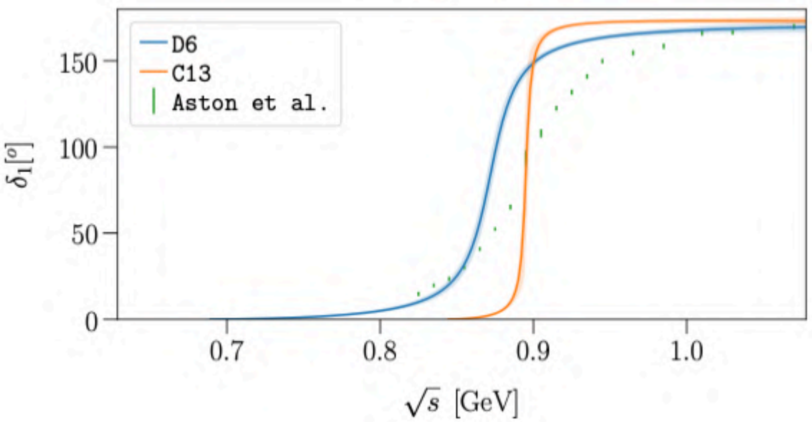
all L,P, Λ=irrep studied

K π scattering: resonances K* ($J^P=1^-$, I=1) and κ (JP =0^+, I=0)

Rendon et al. 2006.14035

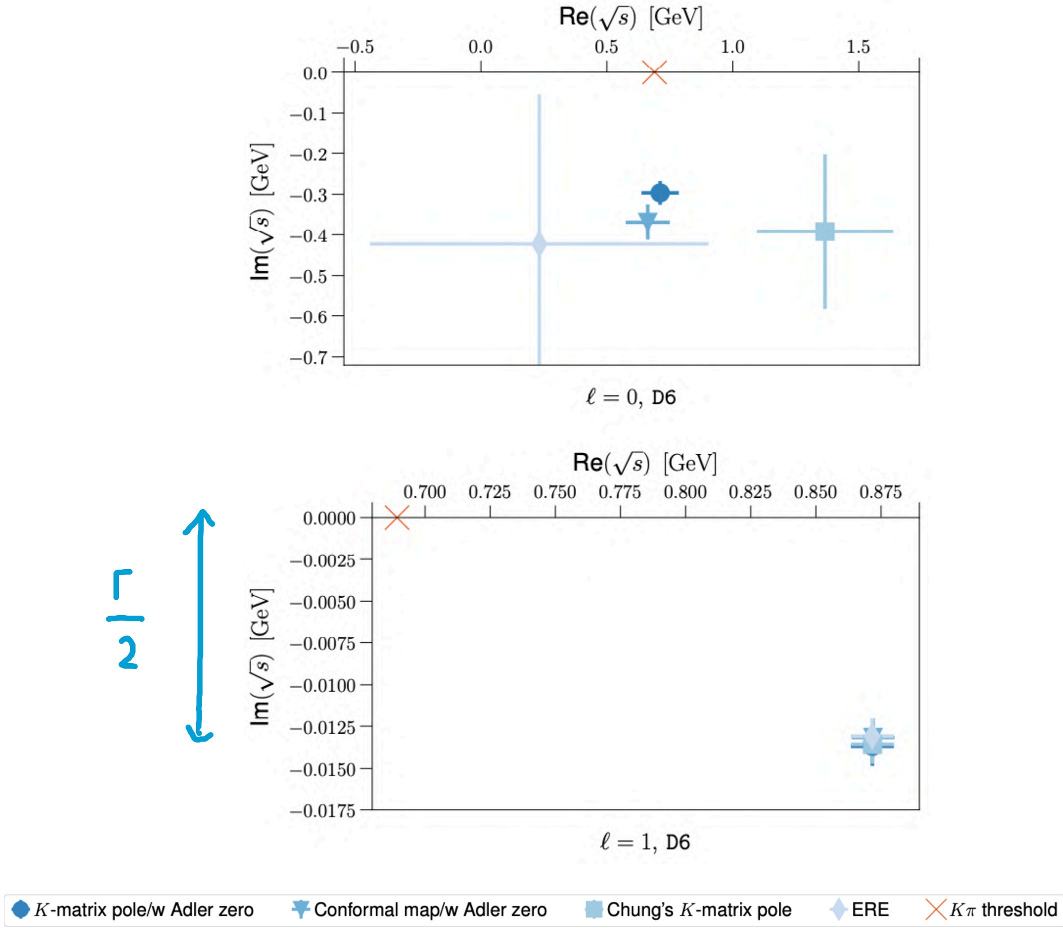
parametrization: effective range expansion





location of poles in the complex E plane m_π=176 MeV

$\frac{\Gamma}{2}$



effective range expansion

blue: m_π=176 MeV

orange: m_π=317 MeV

green: exp

PDG
- $K_0^*(700)$ T-Matrix Pole $\sqrt{s}$ — $(630-730) - i(260-340)$ MeV
- K*(892): — m=892 MeV, Γ=51 MeV

Operators for

two harons with spin zero

Operators for two hadrons with spin 0

meaning of rows

of irrep T1 of Oh (J=1):

x,y,z or mJ=-1,0,1

These transform into each-other

with rotations

$H_{1,2}$: π, K, D, B, η, ... relevant for : $\pi\ \pi$, π K, π D, D D , ... scattering

Apply projection operator to irrep Gamma and row r on arbitrary operator with P=p1+p2

$$O_{\vec{P},\Gamma,r} = \sum_{\tilde{R}\in\Gamma} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R} H_1(\vec{p}_1) H_2(\vec{p}_2) \tilde{R}^{-1} = \sum_{\tilde{R}\in\Gamma} T^{\Gamma}_{r,r}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1) H_2(\tilde{R}\vec{p}_2)$$

creation operatos have T* insted

since hadrons carry no spin in case we consider

$T^{\Gamma}(R)$ = representation matrix of element R in irrep Gamma

$T^{\Gamma}_{rr}(R)$ = diagonal element of T

Proof:

$$\sum_{\tilde{R}\in G} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R} O_{\Gamma',r'} \tilde{R}^{-1} = \sum_{\tilde{R}} T^{\Gamma}_{r,r}(\tilde{R}) \sum_{r''} T^{\Gamma'}_{r'',r'}(\tilde{R})^* O_{\Gamma',r''}$$

$$= \sum_{r''} \Big[\sum_{\tilde{R}} T^{\Gamma}_{r,r}(\tilde{R}) T^{\Gamma'}_{r'',r'}(\tilde{R})^*\Big] O_{\Gamma',r''}$$

Wigner-Eckart orthogonality theoem used

in last step; nG=nmber of elements R

$$= \frac{\dim_{\Gamma}}{n_G} \sum_{r''} \delta_{\Gamma\Gamma'} \delta_{rr''} \delta_{rr'} O_{\Gamma',r''} = \delta_{\Gamma\Gamma'} \delta_{rr'} O_{\Gamma,r}$$

Operators for two hadrons with spin 0

Apply projection operator to irrep Gamma and row r on arbitrary operator with P=p1+p2

$$O_{\vec{P},\Gamma,r} = \sum_{\tilde{R}\in\Gamma} T^{\Gamma}_{r,r}(\tilde{R})\ \tilde{R} H_1(\vec{p}_1) H_2(\vec{p}_2) \tilde{R}^{-1} = \sum_{\tilde{R}\in\Gamma} T^{\Gamma}_{r,r}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1) H_2(\tilde{R}\vec{p}_2)$$

$T^{\Gamma}(R)$ = representation matrix of element R in irrep Gamma

$T^{\Gamma}_{rr}(R)$ = diagonal element of T

$$\chi^{\Gamma}(R) \equiv \sum_{r=1}^{dim(\Gamma)} T^{\Gamma}_{rr}(R)$$ = character

characters of elements in various irreps are often available in chemical web-pages, for example

http://symmetry.jacobs-university.de

example: 1D irrep

$$O_{\vec{P},\Gamma} = \sum_{\tilde{R}\in\Gamma} \chi^{\Gamma}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1) H_2(\tilde{R}\vec{p}_2)$$

Example of 1D irrep: P=(1,1,0)

symmetry group C_{2v} or Dic_2

Leskovec, SP: 1202.2145

$C_n(v)$: rotation by angle 2*pi/n around v

sigma(v): reflection with respect to plane perpendicular to v

names of irreps depend on reference

respresent.	dim	Id	$C_2(e_x+e_y)$	$\sigma(e_x-e_y)$	$\sigma(e_z)$
irred. A_1	1	1	1	1	1
irred. A_2	1	1	1	-1	-1
irred. B_3	1	1	-1	1	-1
irred. B_2	1	1	-1	-1	1

B3 is one-dimensional irrep:

only character appears in projector

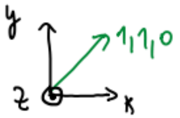
$$O_{\vec{P},\Gamma} = \sum_{\tilde{R}\in\Gamma} \chi^{\Gamma}(\tilde{R})\; H_1(\tilde{R}\vec{p}_1)H_2(\tilde{R}\vec{p}_2)$$

$$O_{P=110,B_3} = \sum_R \chi_{B_3}(R)\; H_1(R(1,0,1))\; H_2(R(0,1,-1)) =$$

$(1,0,1)+(0,1,-1)=(1,1,0)$

$$= H_1(1,0,1)\,H_2(0,1,-1) - H_1(0,1,-1)\,H_2(1,0,1)$$

$R=I,\ \chi=1$ $\qquad$ $R=C_2(1,1,0),\ \chi=-1$

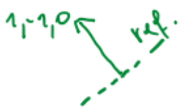


inverts x,y

$z\to -z$

$$+\ H_1(0,1,1)\,H_2(1,0,-1) - H_1(1,0,-1)\,H_2(0,1,1)$$

$R=\sigma(1,-1,0)\quad \chi=1$ $\qquad$ $R=\sigma(0,0,1)\quad \chi=-1$



inverts x,y

z : inv.

leaves x,y inv.

inverts z

This operator can be used to get info on scattering in partial waves l=1 and l=2

(unfortunately both mix in the quantization condition via non-diagonal G)

Example of P=(0,0,0)
irreps of O_h have all more than one 1D

- projection to specific row r

$$O_{\vec{P},\Gamma,r} = \sum_{\tilde{R}\in\Gamma} T^{\Gamma}_{r,r}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1)H_2(\tilde{R}\vec{p}_2)$$

T^{Γ}(R) for all irreps of Oh are listed in App. A of Bernard, Lage, Meissner, Rusetsly 0806.4495

matrices T^{Γ}(R) are not (easily) available for all groups

- characters χ^{Γ}(R) are (easily) available for all groups

if one wants to project just to irrep Γ and does not care to which row r: sum LHS and RHS of equation above over r

$$\sum_r O_{\vec{P},\Gamma,r} = \sum_{\tilde{R}\in\Gamma} \sum_r T^{\Gamma}_{r,r}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1)H_2(\tilde{R}\vec{p}_2)$$

$$O_{\vec{P},\Gamma} = \sum_{\tilde{R}\in\Gamma} \chi^{\Gamma}(\tilde{R})\ H_1(\tilde{R}\vec{p}_1)H_2(\tilde{R}\vec{p}_2)$$

Coupled-channel scattering

most of hadronic resonances decay strongly to several final states

$f_0(980) \to \pi\pi, K\bar{K}$

$a_0(980) \to \pi\eta, K\bar{K}$

$a_1(1260) \to \rho\pi, \sigma\pi, \ldots$

$K_0^*(1430) \to K\pi, K\eta, K\eta'$

$D_3^*(2750) \to D\pi, D^*\pi$

almost all exotic hadrons decay stronly to several final states

$\bar{c}c u\bar{d}: Z_c \to J/\psi\,\pi, D\bar{D}^*, \eta_c\rho, \ldots$

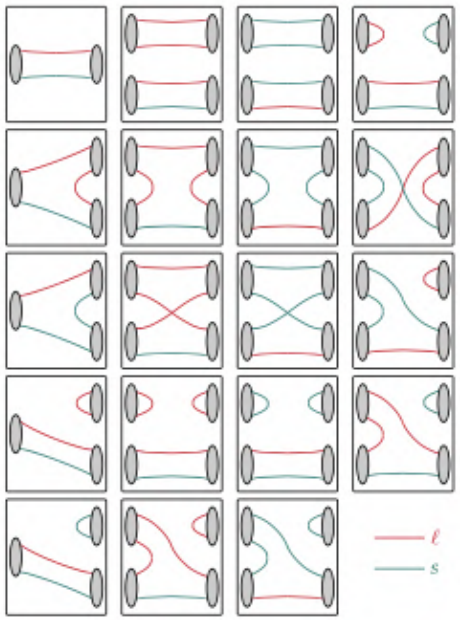
$\bar{b}b u\bar{d}: Z_b \to \Upsilon(1S)\pi, h_b(1P)\pi, B\bar{B}^*, \ldots$

$\bar{c}c uud: P_c \to J/\psi\, p, \Sigma_c D, \ldots$

$\bar{c}c\bar{c}c: X(6900) \to J/\psi\, J/\psi, \eta_c\eta_c, \ldots$

Resonances in Kπ, Kη coupled-channel scattering

Wick contractions



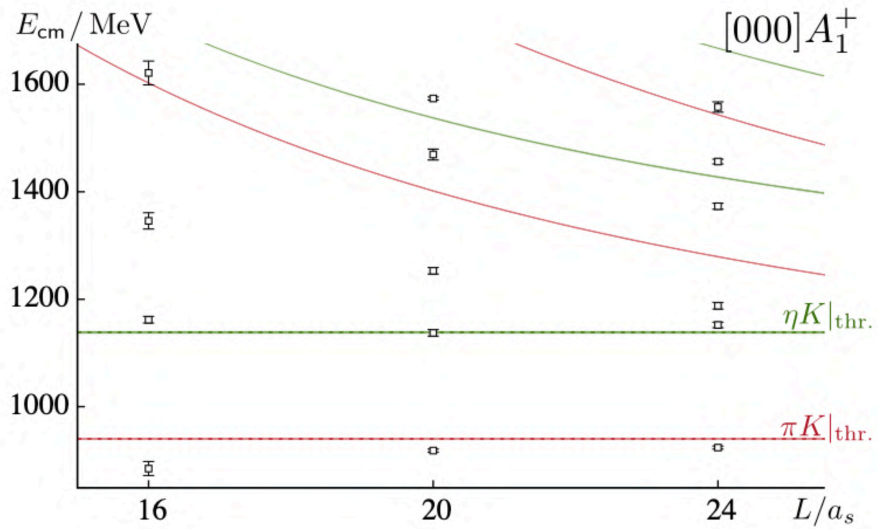
first coupled-channel scattering study:

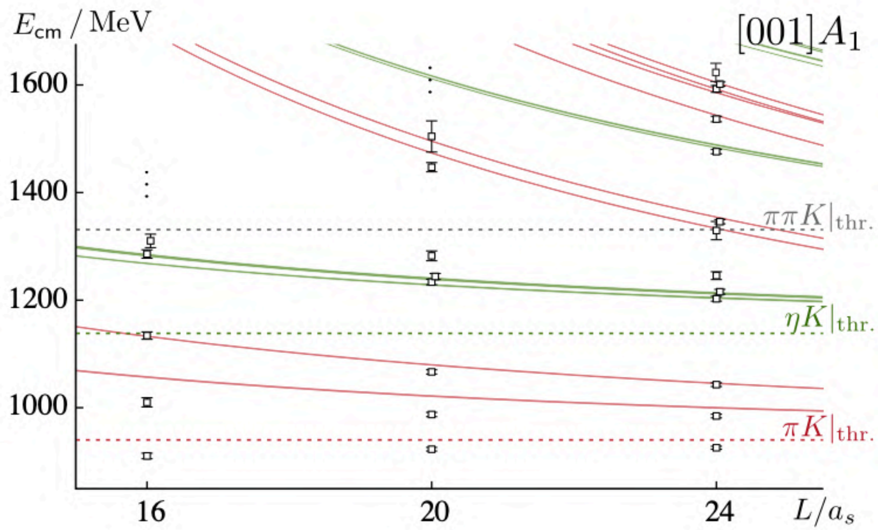
HSC (Wilson, Dudek,Edwards, Thomas): PRL 2014, PRD 2014

$$O\,:\,\bar{q}q,\,K(\vec{p}_1)\pi(\vec{p}_2),\,K(\vec{p}_1)\eta(\vec{p}_2)$$

similar to previouly mentioned study of Kπ,

but with additional channel





Coupled-channel scattering matrix

Consider irrep where only partial wave l contributes (for simplicity)

S is non-diagonal
since $K\pi$ <-> $K\eta$ is possible

one-channel scattering

$SS^+ = I$

$$S = e^{2i\delta}$$

$$\underset{1\times 1}{S} = 1 + i\frac{p}{4\pi E}\underset{1\times 1}{M}$$

two-channel scattering

$SS^+ = I$

$$S = \begin{array}{cc} & \begin{array}{cc} K\pi & K\eta \end{array} \\ \begin{array}{c} K\pi \\ K\eta \end{array} & \begin{vmatrix} \eta\, e^{2i\delta_1} & \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} \\ \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} & \eta\, e^{2i\delta_2} \end{vmatrix} \end{array}$$

$$\underset{2\times 2}{S} = I + i\frac{p}{4\pi E}\underset{2\times 2}{M} \qquad \underset{K\pi}{\delta_1}(E), \underset{K\eta}{\delta_2}(E), \eta(E)$$

Determination of coupled-channel scattering matrix for 2 channels

Consider irrep where only partial wave l contributes (for simplicity); $E=E_{cm}$

$$\det[1 + i\mathcal{M}(E)\mathcal{G}(E)] = 0$$

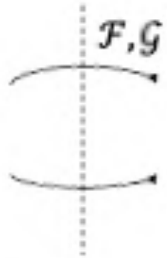
quantization condition

Sharpe & Hansen 1204.0826 and others

$$\begin{array}{ccc} & K\pi & K\eta \\ S = & \left|\begin{matrix} \eta\, e^{2i\delta_1} & \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} \\ \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} & \eta\, e^{2i\delta_2} \end{matrix}\right| & \begin{matrix} K\pi \\ K\eta \end{matrix} \end{array}$$

$$\underset{2\times 2}{\underset{\downarrow}{S}} = I + i\frac{p}{4\pi E}\underset{2\times 2}{\underset{\downarrow}{\mathcal{M}}} \qquad \underset{K\pi}{\underset{\downarrow}{\delta_1(E)}},\ \underset{K\eta}{\underset{\downarrow}{\delta_2(E)}},\ \eta(E)$$

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}_{11} & 0 \\ 0 & \mathcal{G}_{22} \end{pmatrix}$$



G is two-loop function: diagonal

each element same as before

$E=E^{lat}$ for given P, irrep

$$f[\mathcal{M}_{11}(E), \mathcal{M}_{22}(E), \mathcal{M}_{12}(E)] = 0$$

impossible to determine

all three from one equation

rescue: parametrization of M(E)

general idea suggested by

Doring, Meissner, Oset, Rusetsky 1205.4838

$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

quantization condition

M is diagonal for

scat of particles

with S=0

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{12} & \mathcal{M}_{22} \end{pmatrix} \qquad \mathcal{G} = \begin{pmatrix} \mathcal{G}_{11} & 0 \\ 0 & \mathcal{G}_{22} \end{pmatrix}$$

1. parametrize M_{ij} as a function of E or p via some parameters (C)

   for example effective range exp.

$M_{ij}(E)=M_{ij}(E,C)$

2. parameters C chosen such that det[]=0 at $E_{cm} = E^{lat}_{cm,\ n}$

$$\det[1 + i\ \mathcal{M}(E_{cm}, C^i_l)\ \mathcal{G}(E_{cm})]_{L,\vec{P},\Lambda} = 0$$

for all L,P, Λ=irrep studied on the lattice

3. previous point is hard to satify exacly, so one searches for C that satisfy it best by minimizing chi2 below

$$\det[1 + i\ \mathcal{M}(E_{cm}, C^i_l)\ \mathcal{G}(E_{cm})]_{L,\vec{P},\Lambda} = 0 \longrightarrow E^{model}_{cm,\ n}\ (C)$$

$$\chi^2(C) = \sum_{a,b}\ [E^{lat}_{cm,a} - E^{model}_{cm,a}(C)]\ \mathrm{cov}^{-1}_{ab}\ [E^{lat}_{cm,b} - E^{model}_{cm,b}(C)]$$

sum a,b over

all discrete energy levels n

and

all L,P, Λ=irrep studied

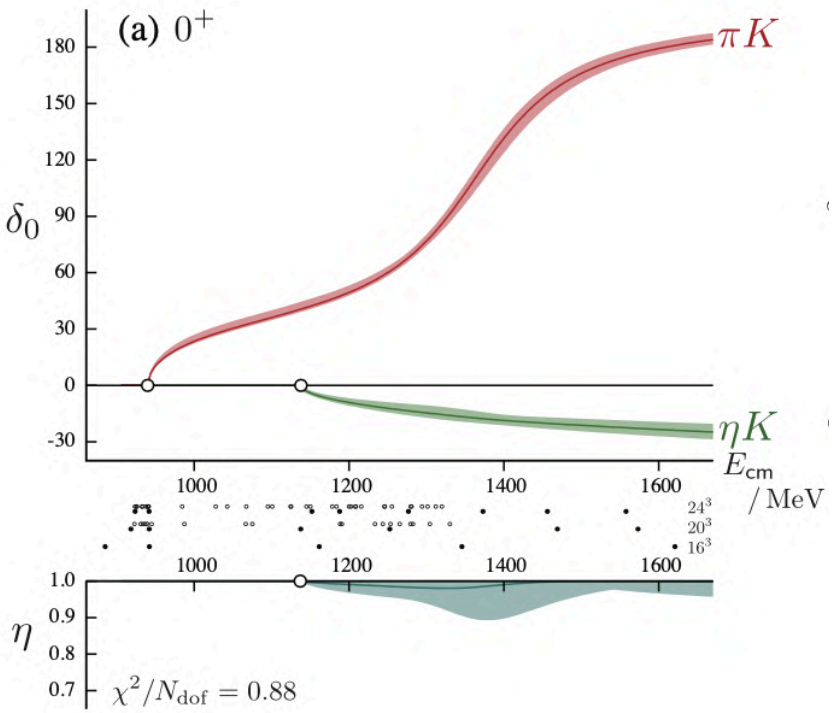
HSC (Wilson, Dudek,Edwards, Thomas):
PRL 2014, PRD 2014

Results for Kπ, Kη scattering in I=0

$$S=\begin{array}{cc} K\pi & K\eta \\ \end{array}$$

$$\begin{array}{c} \\ S= \end{array}\begin{array}{cc} K\pi & K\eta \\ \left|\begin{array}{cc} \eta\, e^{2i\delta_1} & \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} \\ \sqrt{1-\eta^2}\, e^{i(\delta_1+\delta_2)} & \eta\, e^{2i\delta_2} \end{array}\right| \end{array}\begin{array}{c} \\ K\pi \\ K\eta \end{array}$$

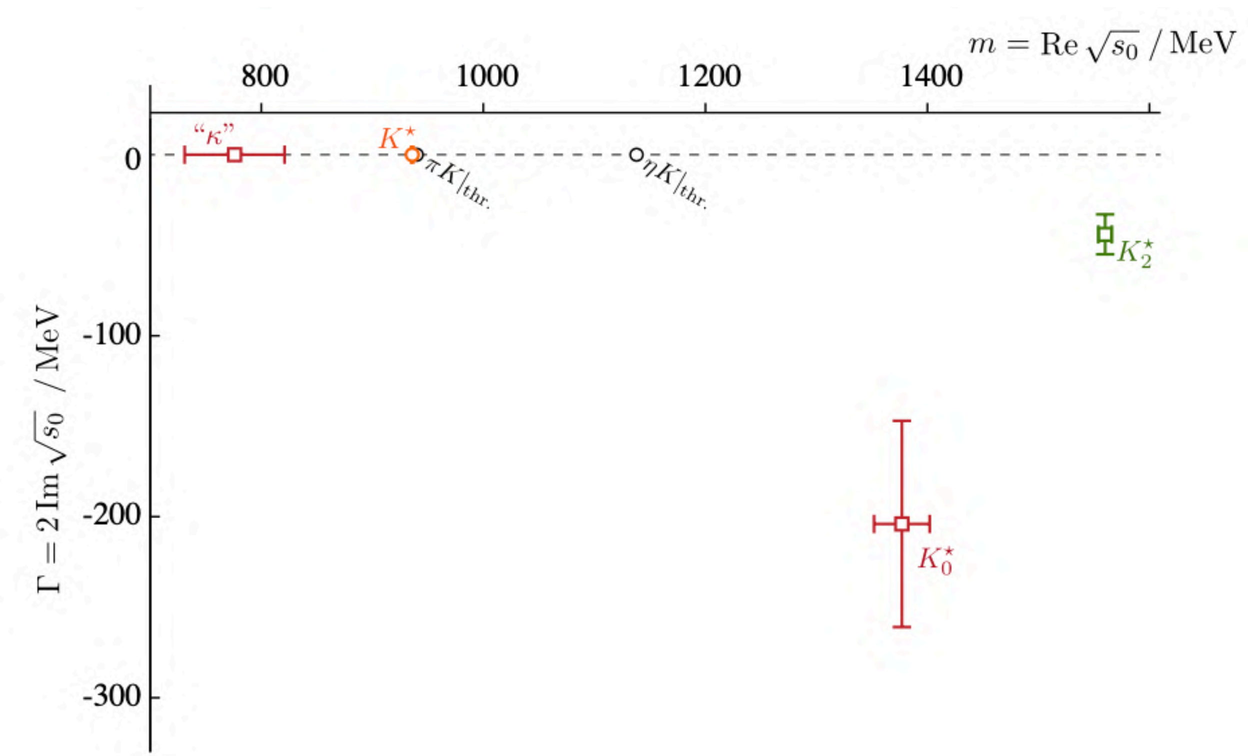
eta=1 : decoupled channels



these two channels are almost decoupeld
for examples of channels that are not decoupeld
see further works by HSC

HSC (Wilson, Dudek,Edwards, Thomas):
PRL 2014, PRD 2014

Locations of poles in Kπ, Kη scattering



● $J^P = 0^+$ □ unphysical sheet (II)

● 1^-

● 2^+ ○ : sheet I

Scattering of particles with spin

Motivation

P=psuedoscalar ($J^P=0^{\wedge -}$) = π , K, D, B, η_c , ...

V=vector ($J^P=1^{\wedge -}$) = D*, B*, J/ψ, Υ_b , B_c^*,... (but not ρ as is unstable...)

N=nucleon ($J^P=1/2^{\wedge +}$) = p, n, Λ, Λ_c, Σ, ... (but not N^* as is unstable...)

All combinations of two-hadron scattering are interesting :

PV: meson resonances and exotics (for example X(3872) in D$\underline{D}$*; Z_c in π J/ψ, D $\underline{D}$* ..)

PN, VN: baryon resonances (e.g. in π N, K N ...) and pentaquarks (e.g. P_c in J/ψ N channel)

NN: nucleon-nucleon and deuterium, baryon-baryon

Several combinations of (S,l) lead to certain J^P

$$N \quad V \xrightarrow{?} J^P = \tfrac{1}{2}^-$$

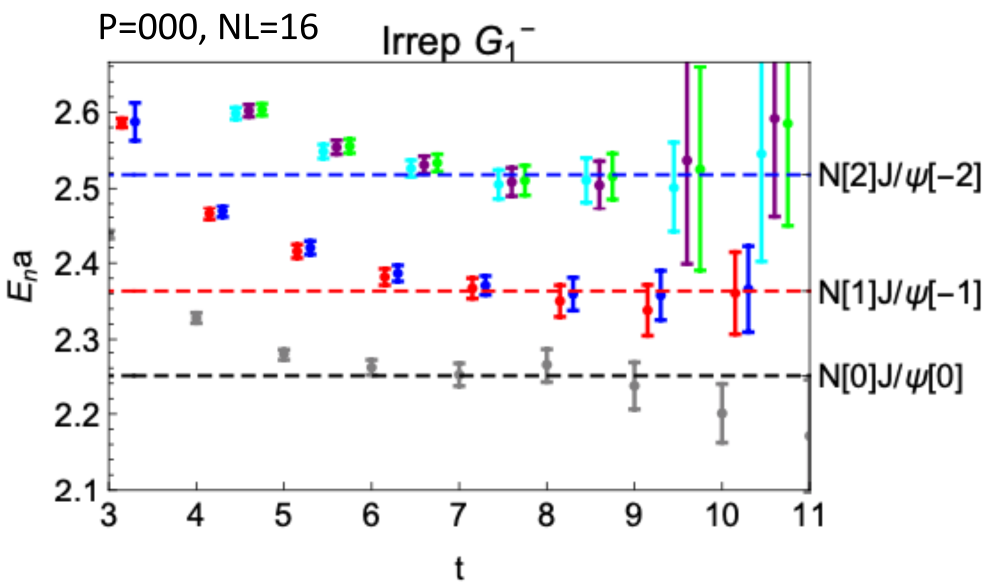
$$\underbrace{\tfrac{1}{2}^+ \quad 1^-}_{S=\frac{1}{2},\,\frac{3}{2}} \qquad l = 0,2$$

$$J^P = \tfrac{1}{2}^- : \quad S = \tfrac{1}{2} \quad l = 0$$

$$S = \tfrac{3}{2} \quad l = 2$$

additional challenge with respect to all previously mentioned

Several nearly-degenerate eigenstates



Skerbis, S.P.

Pc channel: 1811.02285

uudcc (LHCb 2015)

$$P_c \to N\ J/\psi$$

note nearly

degenerate En

$$N \quad V \xrightarrow{?} J^P = \tfrac{1}{2}^-$$

$$\underbrace{\tfrac{1}{2}^+ \quad 1^-}_{S=\frac{1}{2},\,\frac{3}{2}} \qquad \ell = 0,2$$

G_1^-

$J^P = \frac{1}{2}^-$: $S = \frac{1}{2}$ $\ell = 0$

$S = \frac{3}{2}$ $\ell = 2$

$J^P = \frac{7}{2}$: $S = \frac{3}{2}$ $\ell = 2$

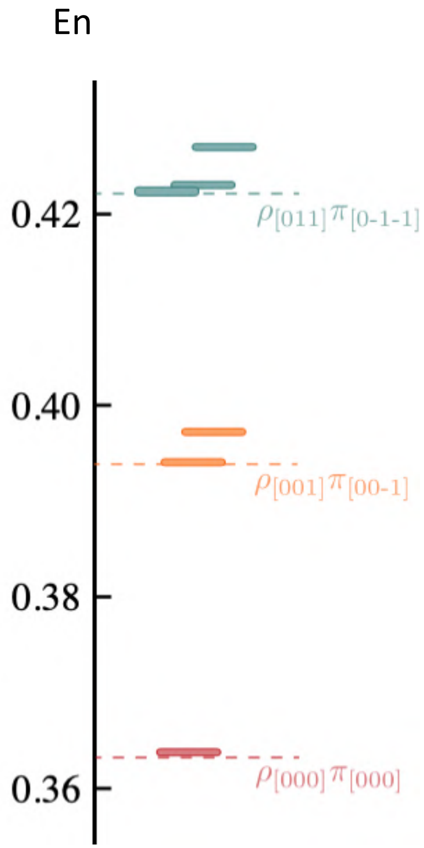
$\rho\ \pi$ scattering ($J^P=1^{\wedge+}$, isospin 2)

Woss et al, HSC
1802.05580

$$\rho\ \pi \xrightarrow{?} J^P = 1^+$$

$$\underbrace{1^-\quad 0^-}_{S=1} \qquad \ell = 0, 2$$

$J^P=1^+$	$S=1$	$\ell=0$
	$S=1$	$\ell=2$



P=000, T1+ irrep, NL=24

note nearly degenerate En

ρ π scattering ($J^P=1^{\wedge+}$, isospin 2)

Woss et al, HSC

1802.05580

$$\underbrace{\underset{1^-}{\rho}\;\underset{0^-}{\pi}}_{S=1} \overset{?}{\longrightarrow} J^P = 1^+ \qquad \ell = 0, 2$$

$J^P=1^+$	$S=1$	$\ell=0$
	$S=1$	$\ell=2$

only J^P is conserved

S and l are not seperately conserved quantum numbers

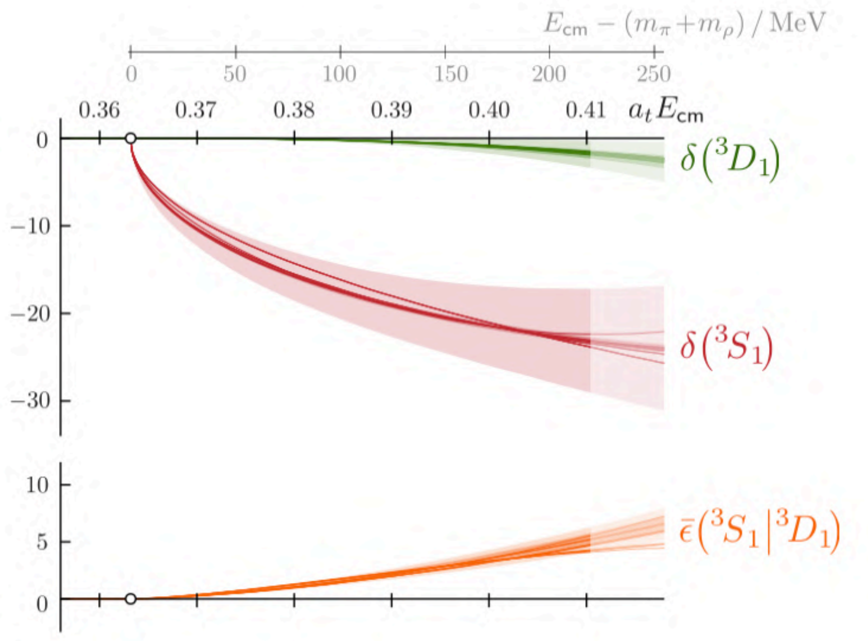
(S,l) can mix (also in continum scattering)

$$\mathcal{M} = \begin{array}{cc} (S,\ell)=(1,0) & (1,2) \\ \left| \begin{array}{cc} \mathcal{M}_{(1,0)\leftrightarrow(1,0)} & \mathcal{M}_{(1,0)\leftrightarrow(1,2)} \\ \mathcal{M}_{(1,0)\leftrightarrow(1,2)} & \mathcal{M}_{(1,2)\leftrightarrow(1,2)} \end{array} \right| \end{array} \begin{array}{c} (1,0) \\ (1,2) \end{array}$$

$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

for two particles with arbitrary spin

Briceno, PRD89, 074507 (2014)



ρ π scattering ($J^P=1^{\wedge +}$, isospin 2)

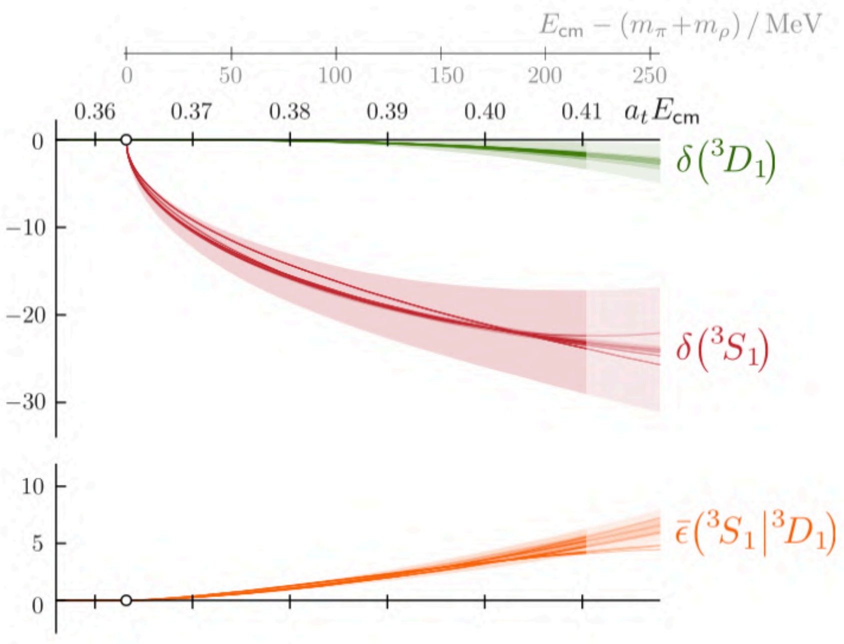
Woss et al, HSC

1802.05580

$$^{2S+1}l_J = ^3 l_1$$

$$\mathbf{t} = \begin{bmatrix} t(^3S_1|^3S_1) & t(^3S_1|^3D_1) \\ t(^3S_1|^3D_1) & t(^3D_1|^3D_1) \end{bmatrix}$$
$$= \frac{1}{2i\rho}\begin{bmatrix} \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3S_1}\right]-1 & i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1}+\delta_{^3D_1})\right] \\ i\sin(2\bar{\epsilon})\exp\left[i(\delta_{^3S_1}+\delta_{^3D_1})\right] & \cos(2\bar{\epsilon})\exp\left[2i\,\delta_{^3D_1}\right]-1 \end{bmatrix} \qquad M \propto E\ t$$

$$\begin{array}{cccc} & (s,l)=(1,0) & (1,2) & \\ \mathcal{M} = & \mathcal{M}_{(1,0)\leftrightarrow(1,0)} & \mathcal{M}_{(1,0)\leftrightarrow(1,2)} & (1,0) \\ & \mathcal{M}_{(1,0)\leftrightarrow(1,2)} & \mathcal{M}_{(1,2)\leftrightarrow(1,2)} & (1,2) \end{array}$$



$$\det[1 + i\mathcal{M}(E_{cm})\mathcal{G}(E_{cm})] = 0$$

for two particles with arbitrary spin

Briceno, PRD89, 074507 (2014)

Scattering of $H_1H_2H_3$

- many resonances have also decay channels $H_1H_2H_3$

- need to consider scattering $H_1H_2H_3$: challenging !!

- generalizations of Luscher's equation, three groups:

  Sharpe, Hansen, Briceno, Romer-Lopez et al ; Rusetky, Hammer et al; Doring, Mai, Alexandru et al

- the only channel considered in QCD: $\pi^+ \pi^+ \pi^+$ (non-resonant repulsive channel)

  Hortz, Hanlon 1911.09047; Hansen et all 2009.04931

- significant development and interest recently

- many challenges left

- not covered in these lectures

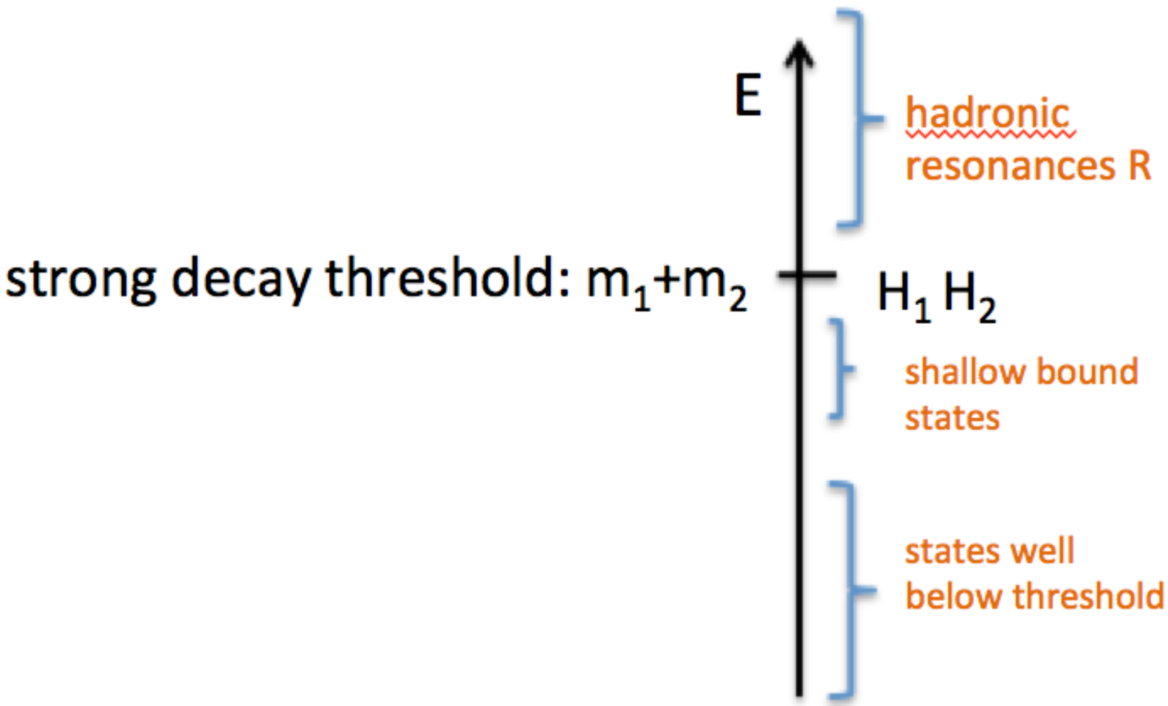
Alternative approach to study scattering on the lattice

- HALQCD approach [for example Aoki et al. 1309.4150 and many other references]
- not covered in these lectures

Summary of four lectures

Clasification of hadron states

these lectures: QCD, no electro-weak interactions, only strong decays



decay strongly

not eigenstates of QCD

can not decay strongly:

p n   D D*   D K

deuteron, X(3872), $D_{s0}^*(2317)$

Mesonic resonance and bound states

- states well below threshold
- strongly decay: resonances
- candidates for shallow bound st.

(analogues of deuterium)

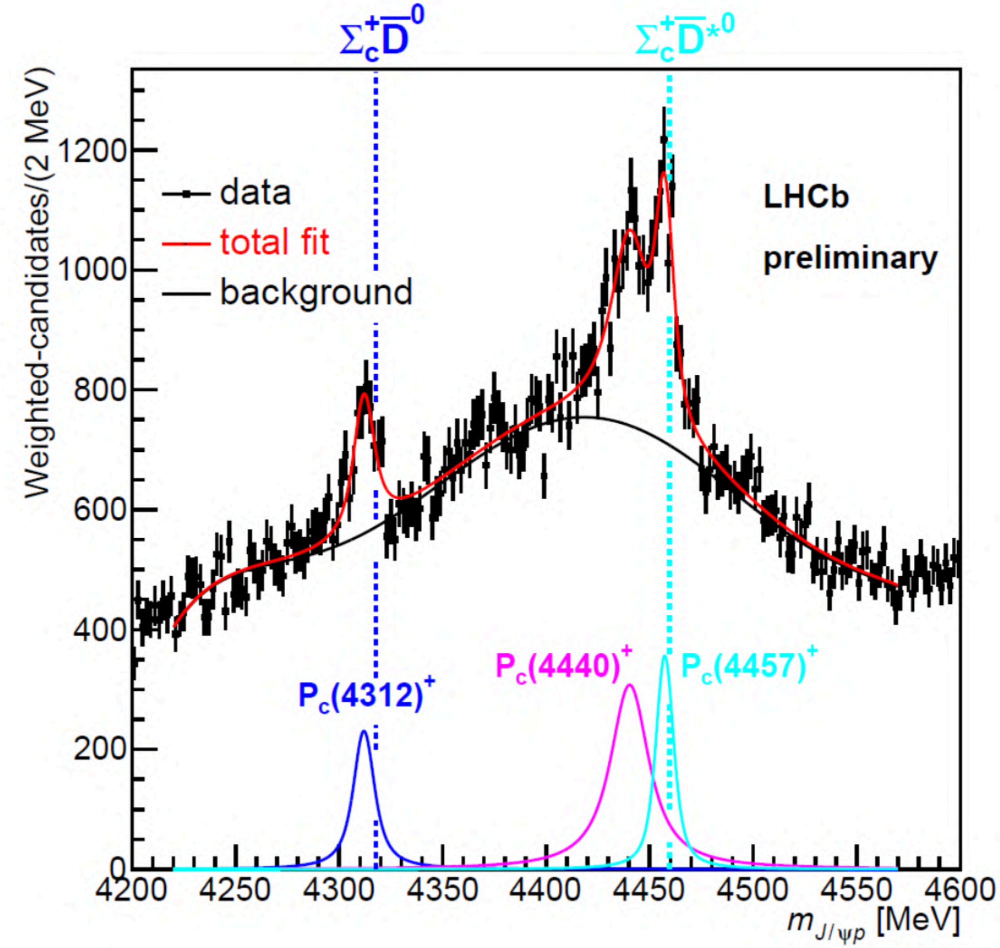
$\bar{u}u$	$\bar{s}u$	$\bar{c}u$	$\bar{c}s$
$\pi^\pm$	$K^\pm$	$D^\pm$	$D_s^\pm$
π^0	K^0	D^0	$D_s^{*\pm}$
η	K_S^0	$D^*(2007)^0$	$D_{s0}^*(2317)^\pm$
$f_0(500)$ or σ was $f_0(60$	K_L^0	$D^*(2010)^\pm$	$D_{s1}(2460)^\pm$
$\rho(770)$	$K_0^*(800)$ or	$D_0^*(2400)^0$	$D_{s1}(2536)^\pm$
$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$	$D_{s2}^*(2573)$
$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$	$D_{s1}^*(2700)^\pm$
$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$	$D_{s1}^*(2860)^\pm$
$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$	$D_{s3}^*(2860)^\pm$
$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$	$D_{sJ}(3040)^\pm$
$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$	
$b_1(1235)$	$K(1460)$	$D(2550)^0$	
$a_1(1260)$	$K_2(1580)$	$D(2600)$	
$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$	
$f_1(1285)$	$K_1(1650)$	$D(2750)$	
$\eta(1295)$	$K^*(1680)$		
$\pi(1300)$	$K_2(1770)$		
$a_2(1320)$	$K_3^*(1780)$		
$f_0(1370)$	$K_2(1820)$		
$h_1(1380)$	$K(1830)$		
$\pi_1(1400)$			
$\eta(1405)$			
$f_1(1420)$			
$\omega(1420)$			
$f_2(1430)$			
$a_0(1450)$			
$\rho(1450)$			

slightly below DK

D*K

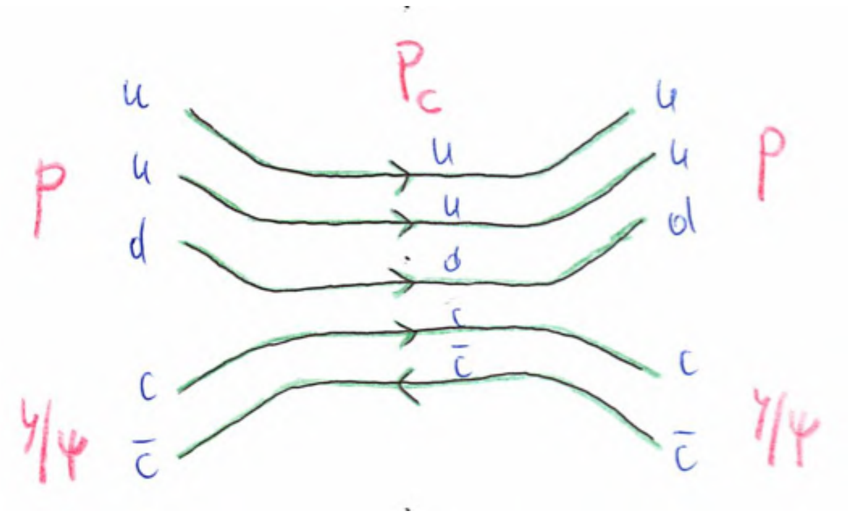
Candidates for exotic hadrons: pentaquarks P_c

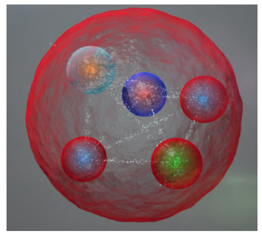
LHCb 2019



$$E(J/\psi\ p)\ [GeV]$$

$$P_c^+ \to J/\psi \quad p$$

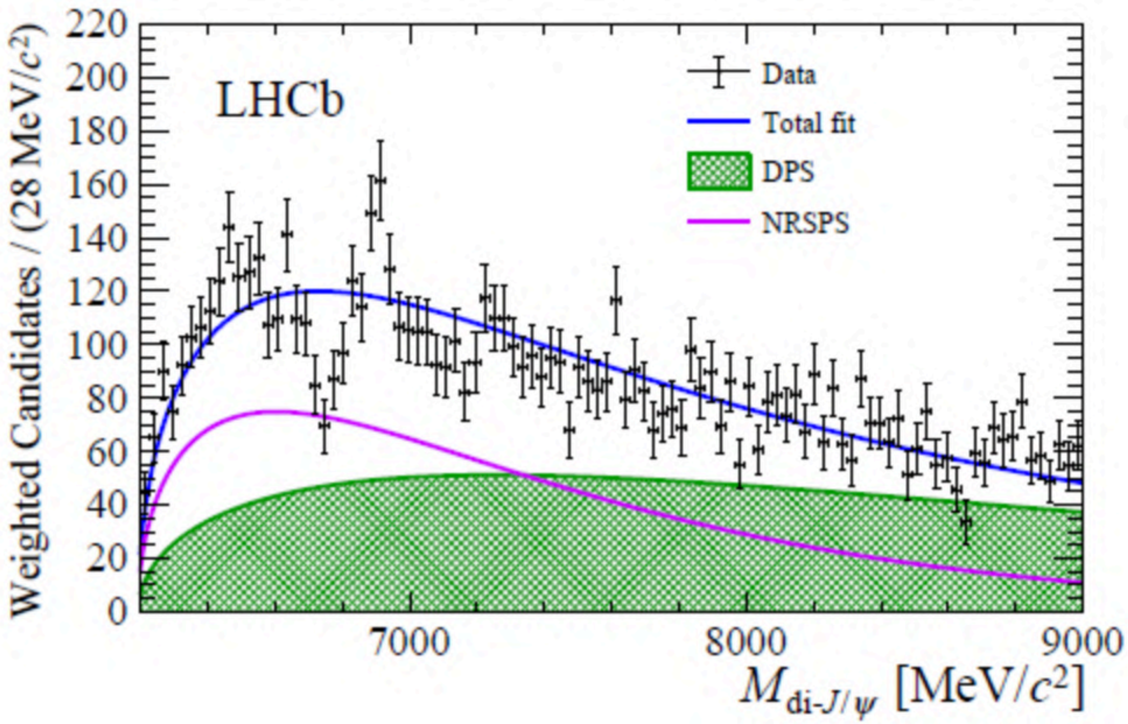




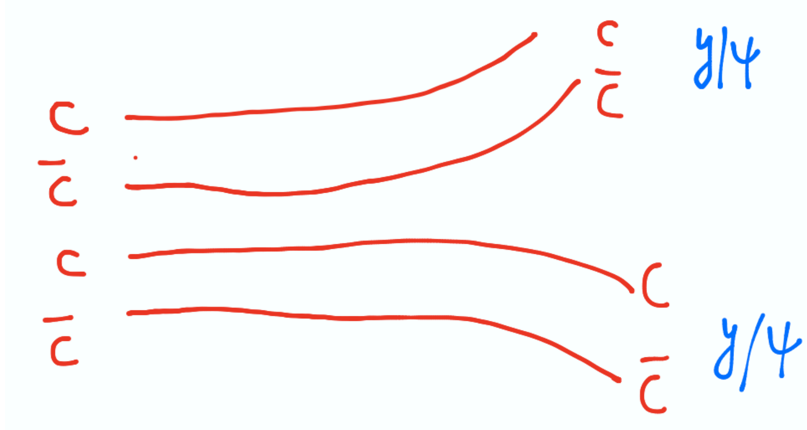
Fully charming tetraquark: $\underline{c}\ c\ \underline{c}\ c$

LHCb 2020

2006.16957



X(6900)

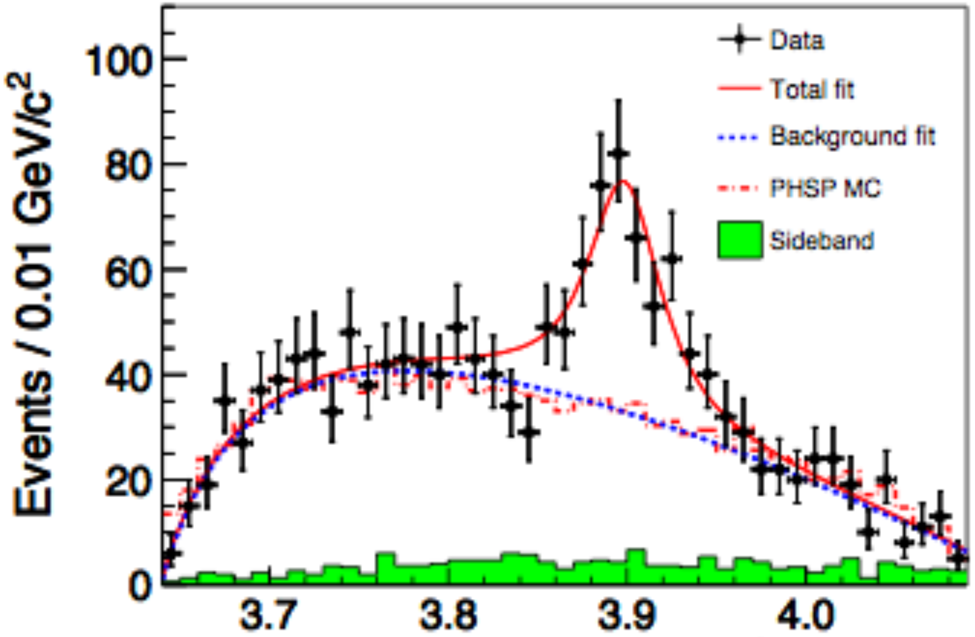


Candidates for exotic hadrons: tetraquarks Z_c^+

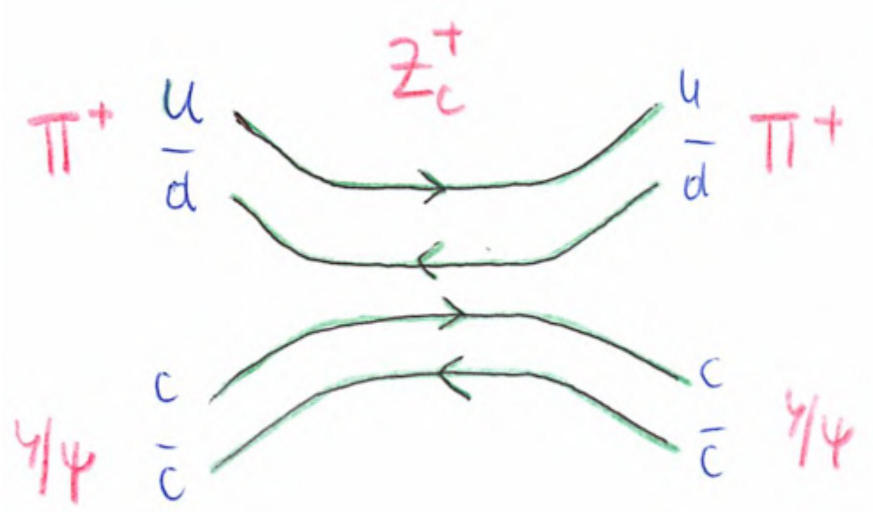
[BESIII & Belle 2013, PRL]

Example: $Z_c^+(3900)$

$$Z_c^+(3900) \rightarrow \; J/\psi \; \pi^+$$



$E(J/\psi\ \pi^+)\ [GeV]$



$$\begin{matrix} \bar{c} & u \\ c & \bar{d} \end{matrix}$$

M ≈ 3900 MeV , Γ ≈ 30 MeV

a number of other exotic hadrons were discovered in past fourteen years ..

Outline, lecture 1

T=scattering amplitude

Scattering in continuum

- shallow bound states, virtual bound states, resonances
- how do we identify them once T is extracted
- poles of T in complex energy-plane, Riemann sheets
- partial waves : l=0, l>0
- near-threshold behavior
- example: spherical well potential

Outline, lecture 2

- Lattice QCD
- reduction of rotational symmetry for the cubic box
- relation of correlation matrices with E_n and $<O_i|n>$
- GeVP variational method to extract E_n and $<O_i|n>$, "proof"
- strongly stable hadrons
- scattering: relation of E_n and scattering amplitude (Luscher's metod)

Outline, lecture 3

- interpolators
- Wick contractions
- all-to-all propagators: distillation method
- Applications:

- bound states
- virtual bound states
- resonances (mostly covered in lecture 4)

In all cases : energy regions where a single channel H1 H2 is present

Outline, lecture 4

- construction of **interpolators** for scattering of two spin-less particles

- **coupled-channel scattering**

- **scattering of hadrons with spin**

Summary

Status with respect to lattice studies of various problems:

mainly solved:

m and Γ of resonances that decay via 1 channel

partly solved :

m and Γ of resonances that decay via 2 or 3 channels (HSC)

m and Γ of resonances that decay to hadrons with spin

unsolved:

resonances that decay to more than 3 channels

resonances that decay to $H_1H_2H_3$

resonances that decay to H_1H_2 and $H_1H_2H_3$

Most interesting exotic hadrons have more than 2 decay channels

most of them have not been rigorously studied on the lattice

many exciting challenges left for the future work

experimental colleagues and phenomenologists are awaiting conclusions from lattice