Charm physics with a tmQCD mixed action approach

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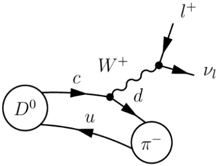
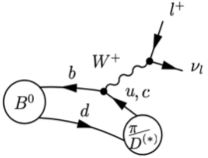
Motivation

- Flavour physics is one of the most promising sectors for the search of physics beyond the Standard Model
- Discrepancies between experiments and SM predictions in the B meson sector deserve particular attention
- A setup that aims at optimising the control of systematic uncertainties in computations involving the heavy quarks is needed
  - for a precise study of charm-quark observables
  - in view of extensions to heavier quark masses

Why do we care?

- Few charm semileptonic results from the lattice
- Interesting to extract CKM matrix elements and heavy quark masses
- Improved experimental results soon





Outline

1. Lattice set-up
   - Sea sector
   - Valence sector
2. Why tmQCD
3. GEVP in the charm sector
   - The generalized eigenvalue problem
   - Testing the GEVP
   - Results

Sea sector

[Lüscher and Schaefer, JHEP 1107 036, 2011; Bruno et al. JHEP 1502 (2015) 043]

We consider the gauge ensembles generated by the CLS initiative

Lüscher-Weisz tree-level improved gauge action

$$S_g[U] = \frac{1}{g_0^2}\left[c_0 \sum_p \mathrm{tr}(1 - U(p)) + c_1 \sum_r \mathrm{tr}(1 - U(r))\right] \quad c_0 = \tfrac{5}{3} \quad c_1 = -\tfrac{1}{12}$$

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$N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions

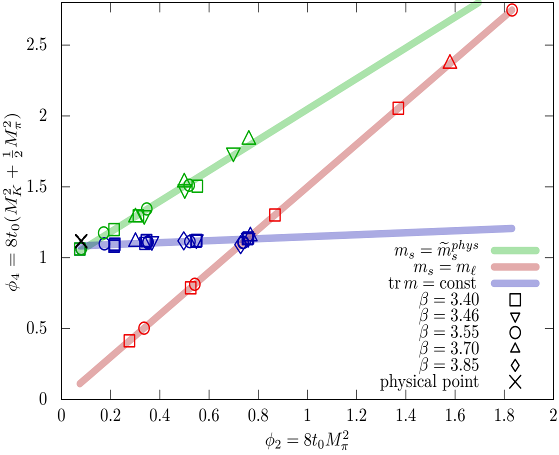
$$S_f[U, \bar{\psi}_f, \psi_f] = a^4 \sum_{f=1}^{3} \sum_x \bar{\psi}_f(x) D_W(m_{0,f}) \psi_f(x)$$

$$D_w(m_0) = \frac{1}{2} \sum_{\mu=0}^{3} \left[\gamma_\mu(\nabla_\mu^\star + \nabla_\mu) - a\nabla_\mu^\star \nabla_\mu\right] + \frac{i}{4} a c_{SW} \sum_{\mu,\nu=0}^{3} \sigma_{\mu\nu} \hat{F}_{\mu\nu} + m_0$$

Sea sector - technical implementation

- Ensembles along lines of constant trace of the bare quark mass matrix

$$\mathrm{tr}\, M_q = 2m_{q,u} + m_{q,s} = \text{const} \qquad \text{where} \qquad m_{q,f} = m_{0,f} - m_{cr}$$



[Plot by J. Simeth]

Lattice spacings :

$a = 0.087, 0.077, 0.065, 0.050, 0.039$ fm

Sea sector - technical implementation

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$$\mathrm{tr}\, M_q = 2m_{q,u} + m_{q,s} = \text{const} \qquad \text{where} \qquad m_{q,f} = m_{0,f} - m_{cr}$$

- Open boundary conditions in time
  - Topological charge flows smoothly [Lüscher and Schaefer, 1206.2809]
  - Avoid topological freezing
  - Reliable estimates for fine values of the lattice spacing

Valence sector [Frezzotti, Grassi, Sint, Weisz, hep-lat/0101001

Frezzotti, Rossi, hep-lat/0306014; Pena, Sint, Vladikas hep-lat/0405028]

Wilson twisted mass Dirac operator

$$D_{tm} = \frac{1}{2}\sum_{\mu=0}^{3}\left[\gamma_\mu(\nabla_\mu^\star + \nabla_\mu) - a\nabla_\mu^\star\nabla_\mu\right] + \frac{i}{4}ac_{SW}\sum_{\mu,\nu=0}^{3}\sigma_{\mu\nu}\hat{F}_{\mu\nu} + \mathbf{m_0} + i\gamma_5\boldsymbol{\mu_0}$$

- Chiral flavour rotation of the fields (twisted basis)

$$\chi = \exp\left(-i\omega\gamma_5\tau^3/2\right)\psi \qquad \bar{\chi} = \bar{\psi}\exp\left(-i\omega\gamma_5\tau^3/2\right) \qquad \psi = \begin{bmatrix} u \\ d \end{bmatrix}, \begin{bmatrix} c \\ s \end{bmatrix}$$

- Maximal twist is achieved by setting $\mathbf{m_0} = \mathbb{1}m_{cr}$, $(\omega = \pi/2)$

$$M = \sqrt{\boldsymbol{m_0}^2 + \boldsymbol{\mu_0}^2} \xrightarrow{\text{full twist}} \boldsymbol{\mu_0} = \text{diag}(\mu_l, -\mu_l, -\mu_s, \mu_c)$$

Advantages of the full twist

- Twisted axial symmetry is only softly broken

$$\partial_\mu \tilde{V}_\mu^a = -2\mu_q \epsilon^{3ab} P^b \quad \Longrightarrow \quad Z_P = \frac{1}{Z_\mu}$$

  pseudoscalar decay constants do not require normalization

- Automatic $O(a)$ improvement for physical observables
  - no parameters tuning
  - residual lattice artefacts of $O(a g_0^4 \,\mathrm{tr}\, M_q)$
  - absence of lattice artefacts proportional to $\boldsymbol{\mu_0}$
- Control of systematics in the heavy quark sector

Advantages of the full twist

[1711.06017, 1812.05458, 1903.00286]

$$m_\pi = m_K = 420 \text{ MeV}$$

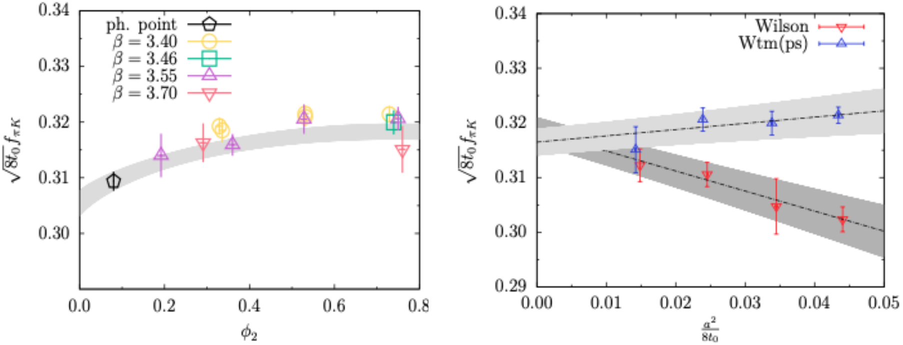

Figure: Continuum limit scaling and extrapolation to the physical point of the quantity $f_{\pi K} = \frac{2}{3}(\frac{1}{2} f_\pi + f_K)$ in units of t_0 using symmetric point ensembles

Advantages of the full twist

[1711.06017, 1812.05458, 1903.00286]

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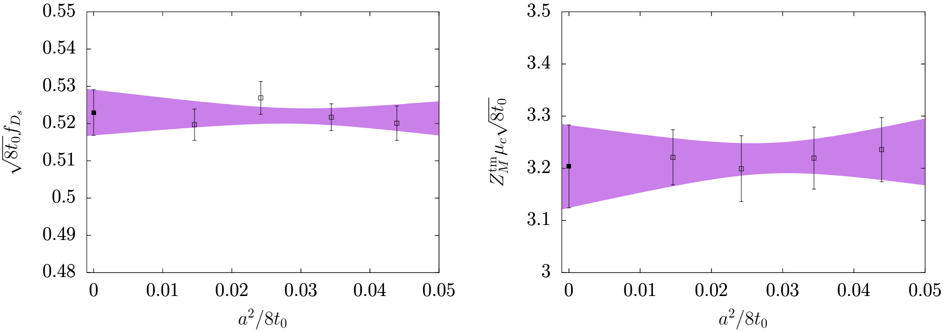

Figure: Continuum limit extrapolation of the decay constant f_{D_s} and renormalized charm quark mass in terms of the reference scale t_0 at the symmetric point $m_u = m_u = m_s$.

Masses and matrix elements in Euclidean QFT

Statement of the problem

Given a local operator $\hat{O}$,

$$C(t) := \langle O(t)O(0)^\dagger \rangle = \sum_{n=1}^{\infty} \psi_n \psi_n^* e^{-E_n t}, \qquad \psi_n = \langle 0 | \hat{O} | n \rangle$$

where $\{|n\rangle\}$ is a complete set of states and $E_n < E_{n+1}$.

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where $\{|n\rangle\}$ is a complete set of states and $E_n < E_{n+1}$.

- We may extract observables by studying the large-time behaviour

$$C(t) \approx \psi_1 \psi_1^* e^{-E_1 t} + \mathcal{O}(e^{-(E_2 - E_1)t})$$

- This may not be easy in practise for several reasons

The Generalized Eigenvalue Problem

[M. Lüscher, U. Wolff, Nucl. Phys. B339 (1990) 222-252]

- Consider a set of operators $\{\hat{O}_i\}$, then combine different interpolators to get a matrix of Euclidean space correlation functions

$$C_{ij}(t) := \langle O_i(t) O_j^\dagger(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n t} \psi_{ni} \psi_{nj}^*, \quad i, j = 1, \ldots, N$$

$$\psi_{ni} \equiv (\psi_n)_i = \langle 0 | \hat{O}_i | n \rangle \quad E_n < E_{n+1}$$

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- The GEVP is defined as

$$C(t) v_k(t, t_0) = \lambda_k(t, t_0) C(t_0) v_k(t, t_0), \quad k = 1, \dots, N, \quad t > t_0$$

Assuming that only N states contribute:

$$\lambda_k^{(0)}(t, t_0) = e^{-E_k (t - t_0)}, \qquad v_k(t, t_0) \rightsquigarrow \psi_m$$

Extraction of observables

[ALPHA, 0902.1265]

- Perturbation theory applied to the truncated N-levels problem gets an estimate of the energies and matrix elements correction terms.

$$E_n^{\text{eff}}(t, t_0) = a^{-1} \log \frac{\lambda_n(t, t_0)}{\lambda_n(t + a, t_0)} = E_n + O(e^{-(E_m - E_n)t})$$

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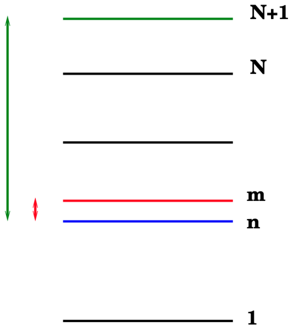
Two efficiency regimes:

- $t > 2t_0$: small gap $E_m - E_n$
- $t \leq 2t_0$: large gap $E_{N+1} - E_n$

$$\Downarrow$$

$$E_n^{\text{eff}} = E_n + O(e^{-\Delta E_{N+1,n}t})$$



Extraction of observables

[ALPHA, 0902.1265]

Thus, for $t \leq 2t_0$:

Energy spectrum

$$E_n^{\text{eff}} = E_n + O(e^{-\Delta E_{N+1,n} t}) \tag{1}$$

Matrix elements

$$\psi_{in}\psi_{jn}^* = e^{E_n t_0} M_{in}(t, t_0) M_{jn}(t, t_0)^* + O(e^{-\Delta E_{N+1,n} t_0}) \text{ at } t - t_0 = \text{fixed}$$

where

$$M(t, t_0) = C(t_0)\tilde{P}(t, t_0) \qquad \tilde{P} := \left(\tilde{v}^{(1)} \dots \tilde{v}^{(N)}\right)$$

GEVP set-up

- We consider the following interpolating fields of different Dirac structures in the twisted basis

$$P = \bar{\psi}\gamma_5\psi \quad A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi \quad V_\mu = \bar{\psi}\gamma_\mu\psi, \quad \mu = 0,\ldots,3$$

$$C_{PP}(t) = \begin{bmatrix} \langle P(t)P(0)\rangle & \langle P(t)A_0(0)\rangle \\ \langle A_0(t)P(0)\rangle & \langle A_0(t)A_0(0)\rangle \end{bmatrix} \quad C_{VV}(t) = \begin{bmatrix} \langle A_k(t)A_k(0)\rangle & \langle A_k(t)V_k(0)\rangle \\ \langle V_k(t)A_k(0)\rangle & \langle V_k(t)V_k(0)\rangle \end{bmatrix}$$

- Then we solve the associated GEVP for the pseudoscalar-pseudoscalar and vector-vector matrix of correlators

$$C_{PP}(t)v_l^P(t,t_0) = \lambda_l^P(t,t_0)C_{PP}(t_0)v_l^P(t,t_0)$$

$$C_{VV}(t)v_l^V(t,t_0) = \lambda_l^V(t,t_0)C_{VV}(t_0)v_l^V(t,t_0)$$

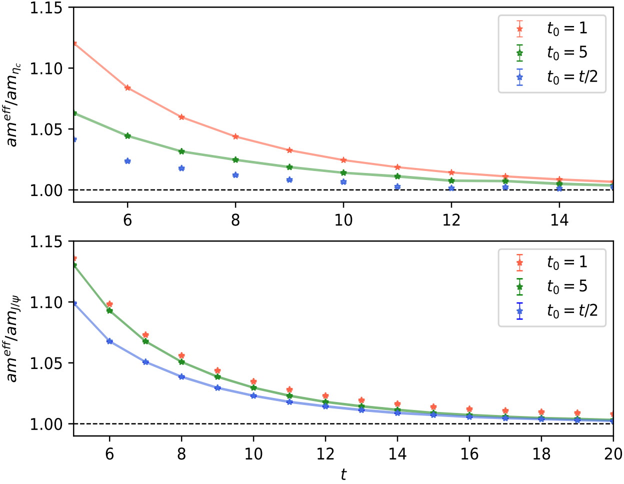
Analysis software

- Julia Programming Language https://julialang.org

- ADerrors.jl by Alberto Ramos https://gitlab.ift.uam-csic.es/alberto/aderrors.jl
  MC data analysis with:
    - Γ-method [Wolff, hep-lat/03060174; Bruno, Sommer, in preparation]
    - Automatic Differentiation [A. Ramos, 1809.01289]
    - $\chi^2_{\text{exp.}}$ fitting routine [Bruno, Sommer, in preparation]

- Juobs.jl by J. Ugarrio and A. Conigli https://gitlab.ift.uam-csic.es/jugarrio/juobs
    - direct reading of mesons-created .dat files
    - construction of observables
    - GEVP implementation

Considerations on the choice of t_0



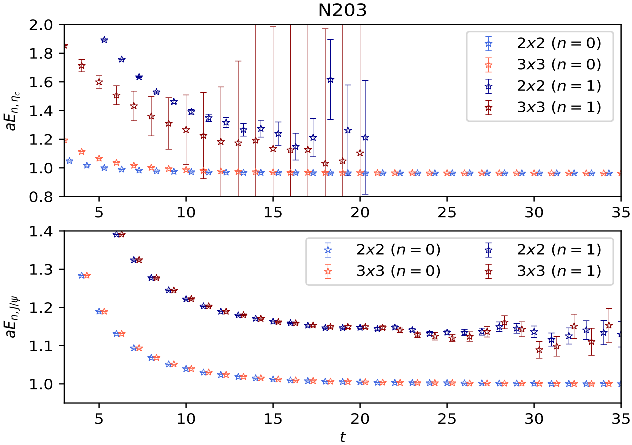
$$\underbrace{\epsilon_n = O(e^{-\Delta E_{m,n} t})}_{t>2t_0}$$

$$\underbrace{\epsilon_n = O(e^{-\Delta E_{N+1,n} t})}_{t\leq 2t_0}$$

Figure: Pseudoscalar and vector masses extracted from the GEVP at different values of t_0

Considerations on the dimension N

- We show the ground and first excited states obtained from a 2×2 and 3×3 GEVP for both the pseudoscalar and vector sectors.



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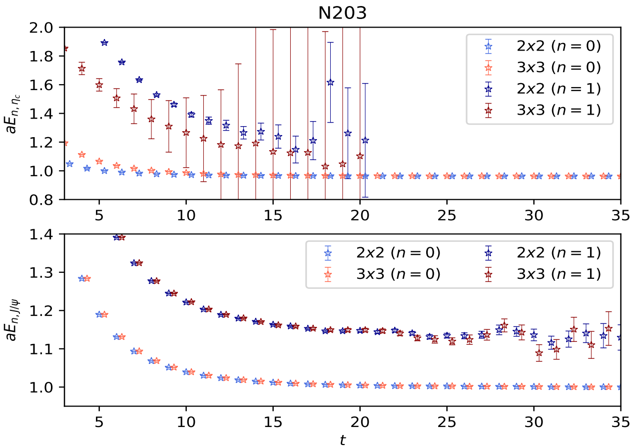

From here on:

- 2×2 GEVP
- $t \leq 2t_0$ regime

Assessing the systematics

[ALPHA, 0902.1265]

- Solving the GEVP gives a series of values for E_n^{eff}, p_n^{eff}.
  An estimate of the systematic error is needed to get final results.

- This is achieved by recalling the large time behaviour at $t \leq 2t_0$:

$$E_n^{\text{eff}} = E_n + \beta e^{-\Delta E_{N+1,n} t}$$
$$p_n^{\text{eff}}(t, t_0) = p_n + \gamma e^{-\Delta E_{N+1,n} t_0}, \qquad t - t_0 = \text{fixed}$$

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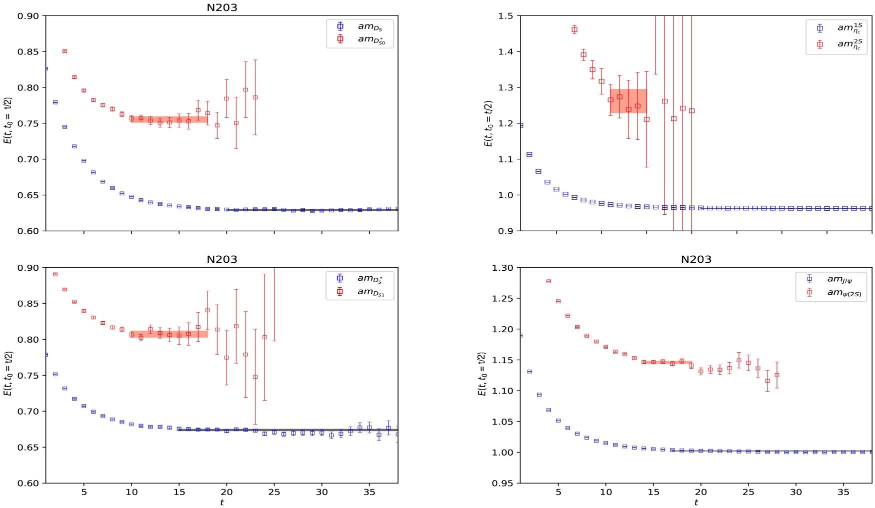
- For a reliable estimate of our quantities of interest, we compute plateau averages of a given observable A by imposing

$$\sigma_A(t_{0,\text{min}}) \geq 4\sigma_A^{\text{syst}}, \qquad \sigma_A^{\text{syst}} \equiv e^{-\Delta E_{N+1,n} t_{0,min}}$$

Pseudoscalar and vector masses

Only connected contributions considered [P. de Forcrand et al., hep/lat0404016]



Pseudoscalar and vector decay constants

Definitions

$$f_{ps} = \sqrt{\frac{2}{m_p^3 L^3}} (\mu_q + \mu_c) |\langle 0 | P^{c,q} | ps \rangle|, \qquad f_v \epsilon_k = Z_A \sqrt{\frac{2}{m_v L^3}} |\langle 0 | A_k | v \rangle|$$

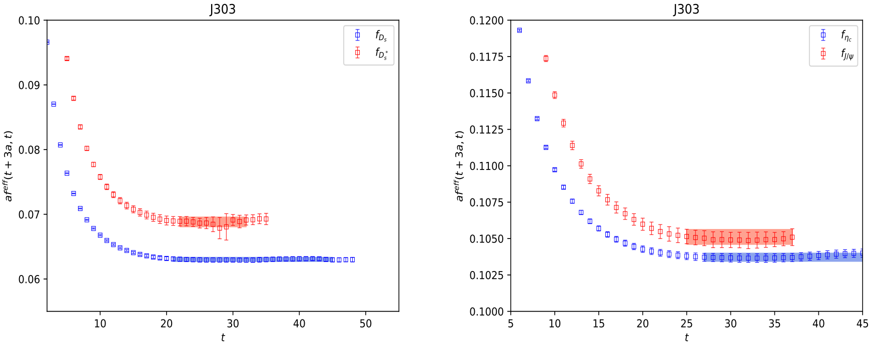
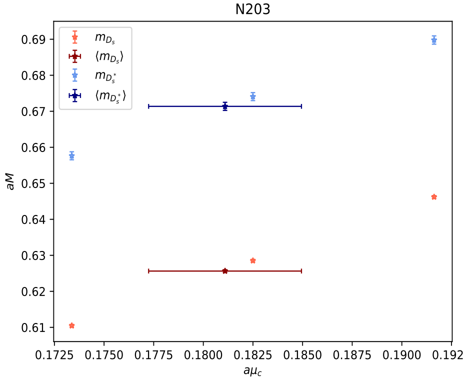
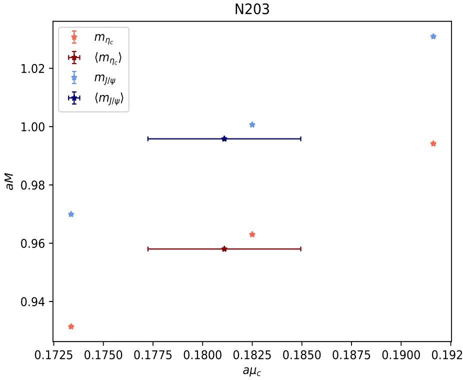


Figure: Pseudoscalar and vector decay constants for charm-light and charmonium sector with proper plateau averages

Matching the charm mass μ_c

- Tune $\mathcal{O}_c = 2/3 m_D + 1/3 m_{D_s} = \mathcal{O}_c^{phys}$
- Compute the desired observables at different values of μ_c around the projected matching point
- Linearly interpolate the results to the matched value μ_c^{match}





Continuum and chiral extrapolation

Renormalization and running of quark mass [ALPHA, 1802.05243]

$$\sqrt{8t_0}O^{\text{cont.}}(a^2/8t_0, \phi_2) = p_1 + p_2 a^2/8t_0 + (p_3 + p_4 a^2/8t_0)\phi_2$$

PRELIMINARY

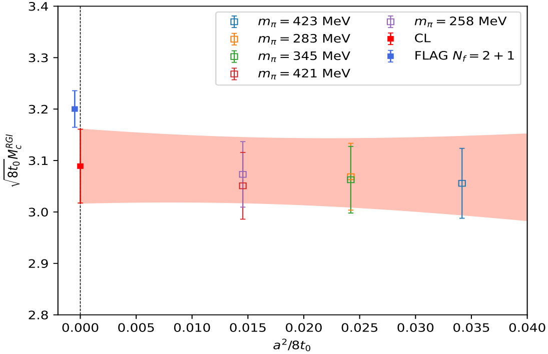


Figure: Continuum limit extrapolation of the renormalized charm quark mass M_c^{RGI} in terms of the reference scale t_0. The charm quark mass is matched by fixing the flavour average to its physical value.

Continuum and chiral extrapolation

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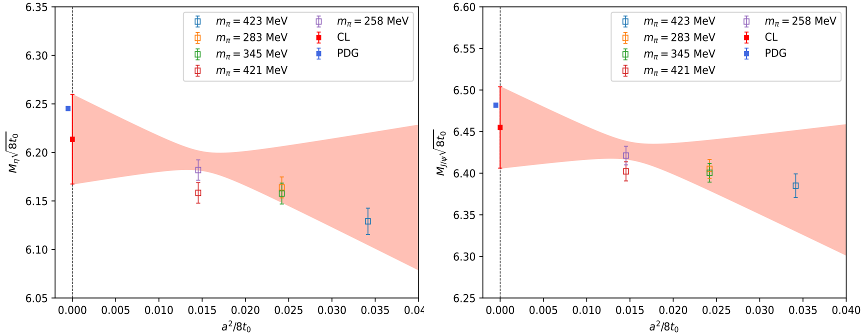
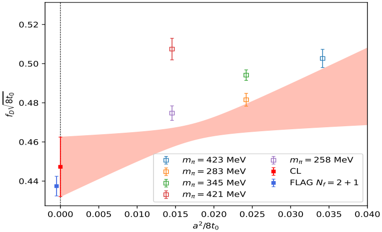
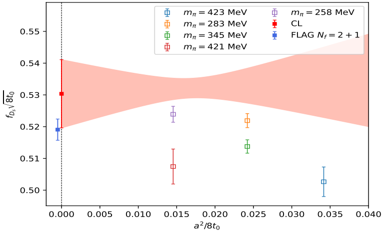
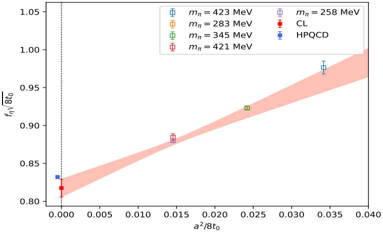


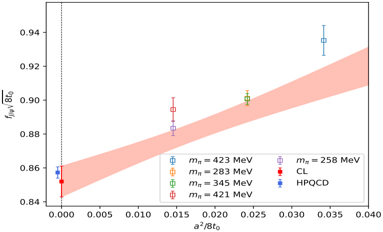
Figure: Continuum limit extrapolation of charmonium M_{η_c} and $M_{J/\psi}$ masses in terms of the reference scale t_0.

Continuum and chiral extrapolation PRELIMINARY









Conclusions and outlook

- We explored GEVP techniques in charm-light and charmonium sector
- We target full control of the main sources of systematic errors
- Extraction of pseudoscalar and vector decay constants relevant for future computation of CKM matrix elements
- Different charm mass matching strategies may be explored
- We will extend this analysis across more ensembles

Thank You!

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More on matrix elements

[Alpha Collaboration, hep/lat0902.1265v2]

Matrix elements are extracted from the effective operator

$$\hat{\mathcal{A}}_n^{\text{eff}}(t,t_0) = e^{-\hat{H}t}\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0), \quad |n\rangle = \hat{\mathcal{A}}^\dagger |0\rangle\,, \quad \hat{H}|n\rangle = E_n |n\rangle$$

$$\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0) = R_n\left(\hat{O}, v_n(t,t_0)\right)$$

$$R_n = (v_n(t,t_0), C(t)v_n(t,t_0))^{-1/2}\frac{\lambda_n(t_0+t/2,t_0)}{\lambda_n(t_0+t,t_0)}$$

Corrections to the large time asymptotic behaviour are parametrized by

$$e^{-\hat{H}t}\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0)^\dagger |0\rangle = |n\rangle + \sum_{n'=1}^{\infty} \pi_{nn'}(t,t_0)\,|n'\rangle$$

More on matrix elements

[Alpha Collaboration, hep/lat0902.1265v2]

They can show that

$$\pi_{nn'} = O(e^{-\Delta E_{N+1,n} t_0}) \qquad \text{at} \;\; t - t_0 = \text{const}$$

Thus, matrix elements of a local operator $\hat{X}$ can be computed via

$$\langle 0| \hat{Q}_n^{\text{eff}} e^{-\hat{H}t} \hat{X} e^{-\hat{H}t} (\hat{Q}_n^{\text{eff}})^\dagger |0\rangle = \langle n| \hat{X} \left|n'\right\rangle + O(e^{-\Delta E_{N+1,n} t_0})$$

And the amplitude between a vacuum and the state $|n\rangle$ is then given by

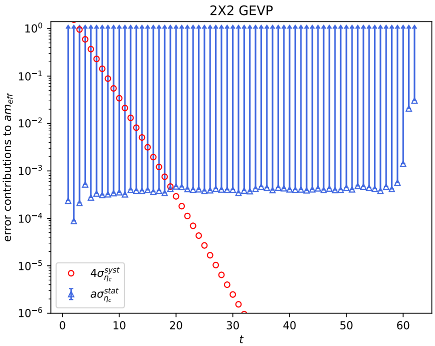
$$p_n^{\text{eff}} = \langle 0| \hat{Q}_n^{eff} e^{-\hat{H}t} \hat{X} |0\rangle = R_n(v_n(t, t_0), C_X), \quad (C_X)_j = \langle O_j(0) X(t) \rangle$$

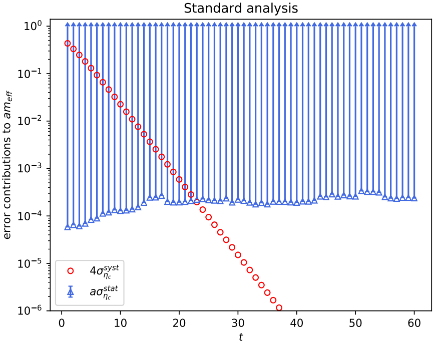
If $\hat{X}$ denotes the time component of an axial current, the decay constant of the associated ground state meson is

$$p_1^{\text{eff}}(t, t_0) = \langle 0| \hat{X} |1\rangle$$

Assessing the systematics

- Statistical error and four times the systematic error contribution due to excited state contaminations to the effective mass am_{η_c}





$\sigma^{\text{syst}} = O(e^{-\Delta E_{2,0} t}), \quad \Delta E_{2,0} \sim 1.5\text{GeV}$

$\sigma^{\text{syst}} = \log \frac{1+ce^{-\Delta E_{1,0} t}}{1+ce^{-\Delta E_{1,0}(t+a)}}$

Considerations on the dimension N

We employ the following interpolators

$$P = \bar{\psi}\gamma_5\psi \quad A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi \quad V_\mu = \bar{\psi}\gamma_\mu\psi, \quad T_{\mu\nu} = \bar{\psi}\gamma_\mu\gamma_\nu\psi$$

to build the GEVP matrices

$$C_{PP}(t) = \begin{bmatrix} \langle P(t)P(0)\rangle & \langle P(t)A_0(0)\rangle \\ \langle A_0(t)P(0)\rangle & \langle A_0(t)A_0(0)\rangle \end{bmatrix} \quad C_{VV}(t) = \begin{bmatrix} \langle A_k(t)A_k(0)\rangle & \langle A_k(t)V_k(0)\rangle \\ \langle V_k(t)A_k(0)\rangle & \langle V_k(t)V_k(0)\rangle \end{bmatrix}$$

$$C_{PP}(t) = \begin{bmatrix} \langle P(t)P(0)\rangle & \langle P(t)A_0(0)\rangle & \langle P(t)V_0(0)\rangle \\ \langle A_0(t)P(0)\rangle & \langle A_0(t)A_0(0)\rangle & A_0(t)V_0(0) \\ \langle V_0(t)P(0)\rangle & \langle V_0(t)A_0(0)\rangle & V_0(t)V_0(0) \end{bmatrix}$$

$$C_{VV}(t) = \begin{bmatrix} \langle T_{kj}(t)T_{kj}(0)\rangle & \langle P(t)A_k(0)\rangle & \langle P(t)V_k(0)\rangle \\ \langle A_k(t)P(0)\rangle & \langle A_k(t)A_k(0)\rangle & \langle A_k(t)V_k(0)\rangle \\ \langle V_k(t)P(0)\rangle & \langle V_k(t)A_k(0)\rangle & \langle V_k(t)V_k(0)\rangle \end{bmatrix}$$